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H-COBORDISMS WITH FOLIATED CONTROL

F. T. FARRELL AND L. E. JONES

ABSTRACT. We announce a foliated version of Ferry's metric h-cobordism theorem [**13**]. Let M be a compact Riemannian manifold and $\mathcal{F}$ a smooth foliation of M such that the sectional curvatures of the leaves of $\mathcal{F}$ are zero. There are numbers $\alpha > 0$ (depending only on $\dim(M)$) and $\delta > 0$ (depending on M and $\mathcal{F}$) so that if W is an h-cobordism over M having δ control in the directions perpendicular to $\mathcal{F}$ and having $\alpha \cdot i(\mathcal{F})$ control in the directions tangent to $\mathcal{F}$, then W is a product cobordism. Here $i(\mathcal{F})$ denotes the greatest lower bounds for the injectivity radii of all the leaves of $\mathcal{F}$.

Statement of theorems. M will denote a smooth compact Riemannian manifold and $\mathcal{F}$ will denote a smooth foliation of M. Note that each leaf L of $\mathcal{F}$ inherits a Riemannian structure from M, and therefore has a well-defined radius of injectivity $i(L)$ (cf. [**5**]). We define $i(\mathcal{F})$ to be the greatest lower bound for all the $i(L)$.

W will denote an h-cobordism with $\partial_- W = M$. W comes equipped with homotopy retractions $h^-: W \times [0,1] \to W$ and $h^+: W \times [0,1] \to W$, satisfying: $h^-(W \times 1) \subset \partial_- W$, $h^-(x,t) = x$ for all $x \in \partial_- W$ and $t \in [0,1]$; $h^+(W \times 1) \subset \partial_+ W$, $h^+(x,t) = x$ for all $x \in \partial_+ W$ and $t \in [0,1]$.

To any continuous path $p: [0,1] \to M$ we can associate two numbers $L_1(p)$, $L_2(p)$ as follows. $L_2(p)$ is the greatest lower bound of all numbers $\lambda > 0$ that satisfy: there is a continuous path $q: [0,1] \to L$ into some leaf L of $\mathcal{F}$ such that $d(q(t), p(t)) \leq \lambda$ for all $t \in [0,1]$ (here $d(\ ,\ )$ denotes the metric on M induced by the Riemannian structure). Define $L_1(p)$ to be the greatest lower bound of all numbers $\lambda > 0$ that satisfy: there is a continuous path $q: [0,1] \to L$ into a leaf of $\mathcal{F}$ such that $d(q(t), p(t)) \leq 2L_2(p)$ for all $t \in [0,1]$; moreover the diameter of $q([0,1])$ in L is less than or equal to λ.

Using the $L_1(\ )$ and $L_2(\ )$ we can now define the diameter of the h-cobordism W in the direction parallel to $\mathcal{F}$—denoted by $D_1(W)$—and in the direction perpendicular to $\mathcal{F}$—denoted by $D_2(W)$. For each $y \in W$ let $p_y^-: [0,1] \to M$ denote the composition

$$[0,1] = (y) \times [0,1] \subset W \times [0,1] \xrightarrow{h^-} W = W \times 1 \xrightarrow{h^-} \partial_- W = M;$$

and let $p_y^+: [0,1] \to M$ denote the composition

$$[0,1] = (y) \times [0,1] \subset W \times [0,1] \xrightarrow{h^+} W = W \times 1 \xrightarrow{h^-} \partial_- W = M.$$

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