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ON EXTENDING SOLUTIONS TO WAVE EQUATIONS ACROSS GLANCING BOUNDARIES

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Introduction. The purpose of this note is to announce some results on the following extension problem. On a C^∞ manifold M with boundary, if u is a given extendible distribution satisfying

$$Pu \in C^\infty(M), \tag{1}$$

under what conditions (on $P, \partial M$, and $u|_{\partial M}$) can u be extended across ∂M as a solution, that is, to a distribution $\tilde{u} \in D'(\tilde{M})$ such that $P\tilde{u} \in C^\infty(\tilde{M})$, for some open manifold $\tilde{M}$ extending M across ∂M? Here P is assumed to be a second-order differential operator on M with smooth coefficients, noncharacteristic with respect to ∂M, and with real principal symbol p having fiber-simple characteristics

$$d_{\text{fiber}} p \neq 0 \quad \text{on} \quad p^{-1}(0) \cap (T^*M \backslash 0) \tag{2}$$

(for example, the wave operator acting in the exterior of a smooth obstacle). After extending the coefficients of P smoothly across ∂M, we can view P as an operator on some open extension $\tilde{M}$ of M.

The problem is easily solved in the two cases where no null bicharacteristics tangent to ∂T^*M are present. When ∂M is everywhere elliptic with respect to P, classical theory implies that the desired $\tilde{u}$ can be found if and only if $u|_{\partial M} \in C^\infty(\partial M)$. When ∂M is everywhere hyperbolic, nothing has to be assumed about $u|_{\partial M} \in D'(\partial M)$, for the extension $\tilde{u}$ can be produced simply by solving the Cauchy problem in a neighborhood of ∂M with Cauchy data given by u. Here we are interested in the two cases where null bicharacteristics tangent to ∂T^*M to first order are present, the diffractive and gliding cases. An example given in [**8**] shows that if the boundary is diffractive, even when $u|_{\partial M}$ is smooth, it may happen that no extension as a solution (in fact, no extension $\tilde{u}$ such that $\rho \notin WF\, P\tilde{u}$ where $\rho \in \partial T^*M$ is a point of null bicharacteristic tangency) exists. Our main result (Theorem 2) implies that, in contrast to the diffractive case, near gliding points extensions as microlocal solutions always exist when $u|_{\partial M}$ is smooth. We construct such an extension after showing that, near a gliding point $\sigma \notin WF\, u|_{\partial M}$, any distribution u satisfying $Pu \in C^\infty(M)$ has the series expansion given in Theorem 1. The proof of Theorem 2 makes essential use of the recent unified treatment of the diffractive and gliding parametrices [**5**], in which the eikonal and transport equations are solved on both sides of the boundary. Full proofs will appear in [**9**].

We proceed to recall some terminology.

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