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Invariance theory, the heat equation, and the Atiyah-Singer index theorem, by Peter B. Gilkey, Mathematics Lecture Series, vol. 11, Publish or Perish, Inc., Wilmington, Delaware, 1984, viii + 349 pp., \$40.00. ISBN 0-914098-20-9

The announcement of what became known as the “Atiyah-Singer Index Theorem” appeared in this Bulletin in 1963. The full proof came out in the Annals of Mathematics in a series of papers between 1968 and 1971, although *Seminar on Atiyah-Singer Index Theorem*, edited by R. Palais, was published in 1965 and contained discussions of the proof and the background needed to understand the theorem. This beginning was typical of the development of the subject. A tremendous amount of research has been generated, yet there has been a relative scarcity of sources for the uninitiated. The book under review is the first detailed exposition of the approach to the index theorem developed by its author and independently by V. K. Patodi in the early seventies. My main complaint about the book is that it is long overdue. All this is partly explained by the heavy demand posed by the subject on both the student and the expository writer. What is required is considerable breadth. Familiarity with analysis, algebraic topology, and Riemannian geometry is an absolute minimum for the student. The subject is in constant flux and has interacted with (this list is certainly incomplete and the ordering is random) number theory, algebraic geometry, mathematical physics, representation theory of Lie groups, probability, and Riemannian geometry.

Index theory is the study of global aspects of systems of linear elliptic partial differential equations. One considers an elliptic operator D between spaces of C^∞ sections of two hermitian vector bundles E and F on a compact Riemannian manifold M . The adjoint operator D^* is also elliptic and, because of the ellipticity, the spaces of solutions of the equations $Du = 0$ and $D^*u = 0$ are finite dimensional. The index of D , $\text{ind}(D)$, is defined as

$$\text{ind}(D) = \dim \ker D - \dim \ker D^*.$$

The celebrated index formula of Atiyah and Singer computes this index as an integral of a *locally* defined expression. More precisely, the integrand is a