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ASYMPTOTIC ENUMERATION AND A 0-1 LAW FOR m-CLIQUE FREE GRAPHS

BY PH. G. KOLAITIS, H.-J. PRÖMEL AND B. L. ROTHSCHILD

In this note we announce some results about the asymptotic behavior of K_m-free graphs. These are the undirected finite graphs which do not contain a complete graph K_m with m vertices (an m-clique) as a subgraph. It is obvious that every graph which contains a clique of size $l+1$ is not l-colorable, and hence has chromatic number at least $l+1$. Also it is well known that there are K_{l+1}-free graphs of arbitrarily large chromatic number. In contrast to this we show that "almost-all" K_{l+1}-free graphs are l-colorable, for any $l \geq 2$. More precisely, we establish

THEOREM 1. *Let $S_n(l)$ be the number of labeled K_{l+1}-free graphs on $\{1,2,\ldots,n\}$ and let $L_n(l)$ be the number of labeled l-colorable graphs on $\{1,2,\ldots,n\}$. Then for any polynomial $p(n)$ there is a constant C such that for all n*

$$S_n(l) \leq L_n(l)\left(1+\frac{C}{p(n)}\right)$$

and hence

$$\lim_{n\to\infty}\left(\frac{L_n(l)}{S_n(l)}\right)=1.$$

The special case of the above theorem for $l=2$ and $p(n)=n$ was proved by Erdös, Kleitman, Rothschild [**1976**], who also showed that

$$\lim_{n\to\infty}\left(\frac{\log L_n(l)}{\log S_n(l)}\right)=1 \quad \text{for any } l \geq 2.$$

In addition to the asymptotic enumeration given by Theorem 1, we derive detailed information about the structure of almost all K_{l+1}-free graphs. We use this to prove that the labeled asymptotic probability of any first-order property on the class $\mathcal{S}(l)$ of all finite K_{l+1}-free graphs is either 0 or 1. C. W. Henson (private communication) obtained the first-order 0-1 law for the class $\mathcal{S}(2)$ from the asymptotic results about K_3-free graphs in Erdös, Kleitman, Rothschild [**1976**]. The classes $\mathcal{S}(l)$ of K_{l+1}-free graphs and $\overline{\mathcal{S}}(l)$ of their complementary graphs occur in the Lachlan-Woodrow [**1980**] characterization of classes of finite undirected graphs having the amalgamation property and closed under induced subgraphs. Together with first-order 0-1 laws already known for other such classes (Fagin [**1976**], Compton [**1984**]) we obtain

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