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IRREDUCIBLE REPRESENTATIONS OF INFINITE-DIMENSIONAL TRANSFORMATION GROUPS AND LIE ALGEBRAS

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1. Introduction. We say that a Lie algebra $\mathfrak{L}$ of vector fields on a smooth manifold M is **transitive** provided that for each point $p \in M$, the vectors $\{X(p): X \in \mathfrak{L}\}$ form the tangent space at p. The algebra $\mathfrak{L}$ is **doubly transitive** if its natural lifting $\mathfrak{L} \oplus \mathfrak{L} = \{X \oplus X: X \in \mathfrak{L}\}$ of $\mathfrak{L}$ to $M \times M$ is transitive on the complement of the diagonal Δ. Higher orders of transitivity are defined analogously. (Just as the full group of diffeomorphisms of a manifold M is n-fold transitive for all n, so is its Lie algebra of vector fields; but the fact about the algebra is far easier to establish.) We are able to exploit the high degree of transitivity of many natural Lie algebras of vector fields to establish irreducibility and inequivalence of certain of their "geometric" or "induced" representations, regarded as unitary representations of the corresponding infinite-dimensional Lie transformation groups. Our technique is a direct descendant of a classical theorem of Burnside on permutation groups.

The applications include much simpler proofs of some of the results of the Soviet school on unitary representations of the group of diffeomorphisms [**10**]. We also get significant generalizations of the pioneering results of Léon van Hove [**9**] on what is now known as "prequantization", i.e., representations of the Poisson bracket Lie algebra of a symplectic manifold. The algebraic technique seems quite fruitful—this is not exactly a surprise—and other applications are forthcoming.

Some of our results were mentioned briefly in [**3**]. Full details will appear elsewhere.

2. The main theorems. If G is a group acting on a discrete set M, then the corresponding representation of G on $l^2(M)$ is irreducible on the orthogonal complement of the constants if and only if the group action on M is doubly transitive. This is essentially due to Burnside (cf. [**2**, p. 249]), and may also be extracted from the work of Mackey [**4**]. The proof is quite simple: If T is an intertwining operator with kernel $k(x, y)$, then we must have the invariance property $k(x, y) \equiv k(x \cdot g, y \cdot g)$ for $x, y \in M$ and $g \in G$. By double transitivity, $M \times M$ decomposes into two G-orbits (the diagonal Δ and its complement) on which the kernel k is constant. Hence k is a linear combination of the

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