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COMPLETE EMBEDDED MINIMAL SURFACES OF FINITE TOTAL CURVATURE

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A long-standing problem in the theory of minimal surfaces is the construction of complete embedded minimal surfaces with finite topology. Many examples of complete properly embedded minimal surfaces in Euclidean three-space have been constructed, but except for the plane, the catenoid, and the helicoid, all the known surfaces have infinite genus. Since it is natural to try to develop a theory of the global geometry of embedded minimal surfaces of finite type, the lack of examples is a major obstacle. In fact, it has often been conjectured that these three examples are the only complete embedded minimal surfaces in $\mathbf{R}^3$ of finite topological type.

In 1980, Jorge and Meeks [**3**] developed a theory to study the topology of complete embedded minimal surfaces in $\mathbf{R}^3$ of finite total curvature. (By a classical theorem of R. Osserman [**4**], a complete minimal surface of finite total curvature is conformally a compact Riemann surface with a finite number of points removed.) They were able to prove that there were no complete embedded minimal surfaces of finite total curvature of genus zero with three, four, or five ends, and, except for the plane, an embedded complete minimal surface of finite total curvature had at least two ends. Recently, R. Schoen [**5**] has proved that the only complete embedded minimal surface of any genus with finite total curvature and two ends is the catenoid. The catenoid has genus zero, two ends, and total curvature -4π.

We have found that there exist embedded minimal surfaces of finite total curvature of every genus.

THEOREM 1. *For every genus $g > 0$ there exists a complete embedded minimal surface M_g of genus g with three ends and finite total curvature $-4\pi(g+2)$.*

Using this theorem we have the following corollary.

COROLLARY. *For every nonnegative integer k, except $k = 2$, there exists a complete embedded minimal surface with total curvature equal to $-4k\pi$.*

PROOF OF COROLLARY. The examples of Theorem 1, together with the plane and the catenoid, give examples of complete embedded minimal surfaces of total curvature $C = -4\pi k$ for every nonnegative integer k except $k = 2$. In [**3**] it is proved that on a complete embedded minimal surface of finite total curvature, $C = -4\pi(g + r - 1)$, where g is the genus and $r \geq 1$ is the number

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