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NONLINEAR ANALOGS OF LINEAR GROUP ACTIONS ON SPHERES

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Contents

Introduction. This article is based upon a principle which is so standard that it is almost a cliché: *The first step to understanding a nonlinear phenomenon is to define and study a suitable linear approximation.* To be more specific, we shall describe some applications of this to the symmetry questions of topological transformation groups.

Given a topological space X, let $\mathrm{Homeo}(X)$ denote the set of self-homeomorphisms of X. This is a group under composition of mappings. If G is an arbitrary group, then a *group action* of G on X is a homomorphism φ: $G \to \mathrm{Homeo}(X)$. Frequently we wish to impose some weak assumptions on φ. For example, if G is a topological group, then we might want φ to have suitable continuity properties. The usual assumption is that the map

$$\mu\colon G \times X \to X, \qquad \mu(g, x) = \varphi(g)[x],$$

is continuous; if G has the discrete topology, then this condition is automatic. Also, it is often convenient to avoid homomorphisms that are in some sense degenerate. For example, every group maps into $\mathrm{Homeo}(X)$ by the constant homomorphism, but for many purposes this trivial sort of group action is uninteresting. The standard procedure is to limit attention to *injective* homomorphisms (= *effective group actions*) unless stated otherwise.

Smooth actions. If X is in fact a differentiable manifold with smooth structure (say) $\mathscr{F}$, it is often useful to consider group actions that are *smooth* in an appropriate sense. By this we mean that G is a Lie group, φ maps G into the

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