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BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 10, Number 2, April 1984
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0273-0979/84 $1.00 + $.25 per page

Geometrical methods in the theory of ordinary differential equations, by V. I. Arnold, Grundlehren der mathematischen Wissenschaften, Volume 250, Springer-Verlag, New York, 1983, x + 334 pp., $36.00. ISBN 0-3879-0681-9

> Among all mathematical disciplines the theory of differential equations is the most important.
> S. Lie (1895)

> [The work of] Smale... shows that the problem of the complete topological classification of differential equations with high dimensional phase space is hopeless...
> V. Arnold (p. 87)

> One picture is worth a thousand symbols.
> Old proverb

Poincaré drew an analogy between algebraic and differential equations. In solving an algebraic equation one first does a *qualitative* investigation, determining the number of real roots by Sturm's theorem; then one carries out the *quantitative* step of numerically evaluating the roots. Similarly with the study of algebraic curves: only after the qualitative step of determining which branches are closed or infinite does one numerically find a certain number of points on the curve. It is the same with differential equations: before numerically evaluating the solution, first one should perform a qualitative investigation into the general form of the solution. Is it bounded or unbounded? Does it oscillate, or converge, or neither? Is it stable or unstable? This last question involves looking at not just a single solution, but all the solutions. In connection with this, Hadamard suggests another parallel with algebraic equations: great progress was made only after Galois and others began to look at the relations between *all* the roots of a polynomial.

The essence of Poincaré's "qualitative" investigations, according to Hadamard, is to regard the values of the unknown function not as a function of the independent variable (usually interpreted as time), but rather as a function of the initial data. The more recent notion of "dynamical system" is an abstract formulation of this point of view.