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A p-ADIC REGULATOR PROBLEM IN ALGEBRAIC K-THEORY AND GROUP COHOMOLOGY

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Let $\mathcal{O}$ be the ring of integers in a number field F. Let $\mathfrak{p} \subset \mathcal{O}$ be a prime ideal and $\mathcal{O}_{\mathfrak{p}} = \varprojlim \mathcal{O}/\mathfrak{p}^s$ be the $\mathfrak{p}$-adic completion of $\mathcal{O}$. Let

$$\hat{K}_n(\mathcal{O}) = K_n(\mathcal{O}) \quad \text{mod torsion},$$
$$\hat{K}^c_n(\mathcal{O}_{\mathfrak{p}}) = K^c_n(\mathcal{O}_{\mathfrak{p}}) \quad \text{mod torsion},$$

where $K_n(\mathcal{O})$ is the algebraic K-theory of Quillen [**Q**] and

$$K^c_n(\mathcal{O}_{\mathfrak{p}}) = \varprojlim K_n(\mathcal{O}/\mathfrak{p}^s)$$

is the "continuous" or "p-adic" algebraic K-theory of $\mathcal{O}_{\mathfrak{p}}$ studied in [**W1**] by Milgram and the author. Results of [**B**] and [**W1**] suggested asking whether

$$\Phi_{\mathfrak{p}} : \hat{K}_n(\mathcal{O}) \to \hat{K}^c_n(\mathcal{O}_{\mathfrak{p}}) \tag{1}$$

or

$$\Phi : \hat{K}_n(\mathcal{O}) \to \bigoplus_{\mathfrak{p}|p} \hat{K}^c_n(\mathcal{O}_{\mathfrak{p}}) \tag{2}$$

is injective, where p is a fixed rational prime and $n > 1$ is odd. Observe that each $\Phi_{\mathfrak{p}}$ is clearly injective for $n = 1$, because $K_1(\mathcal{O}) = \mathcal{O}^*$ and $K^c_1(\mathcal{O}_{\mathfrak{p}}) = \mathcal{O}^*_{\mathfrak{p}}$. A much harder problem is whether $\Phi \otimes \mathbf{Z}_p$ is injective. For $n = 1$ and F totally real abelian, injectively of $\Phi \otimes \mathbf{Z}_p$ on the subgroup of $\mathcal{O}^*$ consisting of those elements congruent to $1 \bmod \mathfrak{p}$ for each $\mathfrak{p} \mid p$ is equivalent to nonvanishing of the p-adic regulator [**Br**, **C**]. As an example of (1) let F be quadratic imaginary. Then is

$$\Phi_{\mathfrak{p}} : \mathbf{Z} \cong \hat{K}_3(\mathcal{O}) \to \hat{K}^c_3(\mathcal{O}_{\mathfrak{p}}) \cong \mathbf{Z}_p \tag{3}$$

injective when $p = \mathrm{char}(\mathcal{O}/\mathfrak{p})$ is unramified with $\mathcal{O}_{\mathfrak{p}} \cong \mathbf{Z}_p$? J.-P. Serre asked an equivalent cohomological version of (1) and (2) prior to the circa 1975 K-theory formulation. For special case (3) injectivity is equivalent to showing $\Phi_{\mathfrak{p}} \otimes Q_p$ is an isomorphism, which in turn amounts to showing

$$Q_p \cong H^3_c(\mathrm{SL}_n(\mathcal{O}_{\mathfrak{p}}); Q_p) \to H^3(\mathrm{SL}_n(\mathcal{O}); Q_p) \cong Q_p \tag{4}$$

is an isomorphism for n large. H^3_c denotes the continuous cohomology of the p-adic group $\mathrm{SL}_n(\mathcal{O}_{\mathfrak{p}})$ and H^3 is the Eilenberg-Mac Lane cohomology of the discrete group $\mathrm{SL}_n(\mathcal{O})$. Compare [**L**]. Numerous examples of (4)

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