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Differential operators for partial differential equations and function theoretic applications, by K. W. Bauer and S. Ruscheweyh, Lecture Notes in Mathematics, Vol. 791, Springer-Verlag, Berlin and New York, 1980, v + 258 pp., $18.00. ISBN 0-3870-9975-1

It has been known for a long time, in a general qualitative way, that solutions of linear partial differential equations of elliptic type, particularly those arising in mathematical physics, share many of the properties of analytic functions of a complex variable. Rendering this phenomenon quantitative and analytic has been a relatively modern development that began in the 1930s with the work of S. Bergman and I. Vekua. Since then it has grown to encompass PDE's of elliptic, parabolic and hyperbolic types into an active field of research referred to as "Function Theoretic Methods in Partial Differential Equations". The general idea is to build a function theory for a specified equation from one that is known a priori. A typical antecedent is analytic function theory, where there is an extensive literature concerning properties such as singularities, analytic continuation, approximation and interpolation, and boundary values.

The development of a function theory for an equation begins by finding an invertible operator that represents a solution as the transform of a unique function called an associate or generator. Analysis of the operator, the associate, and the inverse operator serves to "transplant" the function theory. Because of this transform dependence, it may not be possible to detail specific properties of a solution from those of the associate; for example, singularities from singularities, interpolates from interpolates, and so on. Therefore, alternate representations are sought as effective supplements. These may, in turn, extend the domain to other classes of problems. Integral operator methods have been efficient in function-theoretic studies, and there are relevant books listed as references.

In 1915 G. Darboux introduced differential operators in connection with the Euler equation. But, a function-theoretic approach to PDE's based on transforms that are differential operators did not emerge until the 1960s and to this the authors have contributed significantly. Recently interest in this direction has been increasing because it permits detailed investigations of problems with physical applications. And, subsequently the related theory had grown to the point where a basic reference was needed to outline its development. The book provides this. In addition, one finds that the comprehensive references and inclusion of previously unpublished work support a sense of direction and overview of the "state of the art" for the differential operator method.

The book is focused on three elliptic PDE's and emphasizes the interplay between the associated function theories and physical applications. Transform equivalent problems arise, as well as "miscellaneous" problems which are