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## ASYMPTOTIC BEHAVIOUR OF EISENSTEIN INTEGRALS

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Let $G$ be a noncompact connected real semisimple Lie group with finite centre. The asymptotic behaviour of Eisenstein integrals associated with a minimal parabolic subgroup of $G$ has to a large extent been studied by Harish-Chandra (unpublished work, see [**12**] for an account, and later in a more general setting in [**5–7**]). Other references are [**9** and **10**]. Harish-Chandra's work depends heavily on a detailed study of systems of differential equations satisfied by these integrals. In [**1**] it is shown that these systems can be transformed into complex differential equations of the regular singular type; the asymptotic behaviour of their solutions is studied by essentially applying the classical Frobenius theory.

In this announcement we present some results obtained by using another classical method, namely the representation of solutions of such equations by compact complex contour integrals (for the hypergeometric equation this method goes back to [**8**]). These integral representations can serve as the starting point for estimation by application of the method of steepest descent. This is closely connected with the use of the method of stationary phase in [**2**], where the asymptotic behaviour of Eisenstein integrals with respect to the spectral variable is studied.

I would like to thank Professor J. J. Duistermaat for suggesting this problem and for the many stimulating discussions we had.

Let $G = KAN$ be an Iwasawa decomposition. Let $\mathfrak{G}$ and $\mathfrak{A}$ be the Lie algebras of $G$ and $A$, $\Delta$ the root system of $(\mathfrak{G}, \mathfrak{A})$, $\Delta^+$ the set of positive roots corresponding to $N$; let $\Delta^{++} = \{\alpha \in \Delta^+; \frac{1}{2}\alpha \notin \Delta^+\}$ and let $\kappa\colon G \to K$, $H\colon G \to \mathfrak{A}$ be defined by $x \in \kappa(x) \exp H(x) N$ $(x \in G)$. Moreover, let $\tau_1, \tau_2$ be two mutually commuting representations of $K$ in a finite dimensional complex linear space $V$ (for convenience of notation we let them both act on the left). Let $M$ be the centralizer of $\mathfrak{A}$ in $K$ and set $V_M = \{v \in V; \tau_1(m)\tau_2(m)v = v\,(m \in M)\}$. For $\lambda \in \mathfrak{A}_c^*$ (the complexified dual of $\mathfrak{A}$), the

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