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ON FIXED POINTS OF AUTOMORPHISMS OF FINITELY GENERATED FREE GROUPS

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ABSTRACT. If ϕ is an automorphism of a finitely generated free group, then $\mathrm{Fix}(\phi)$ is finitely generated.

0. Introduction. A conjecture attributed to G. P. Scott [**St2**] states that the fixed points of an automorphism ϕ of a finitely generated (fg) free group F is fg. It is known that this is the case if ϕ is periodic [**D-S**] or geometric [**J-S**], where ϕ is called geometric if it is the induced map on π_1 of a homeomorphism of (M, x), where M is a compact bounded two-dimensional manifold and $x \in M$. In a recent article [**G2**] we showed that Scott's conjecture is valid for automorphisms of π_1 of a finite graph induced by change of maximal trees.

In this article we announce the affirmative solution of Scott's conjecture. The methods involved are an extension of methods of [**G2**] and blend combinatorics, topology, and group theory.

1. Graphs. We work with a combinatorial notion of a graph introduced in [**G1**] (and based on an idea of Serre's [**S**]). A graph X is a nonempty set with involution $x \to \bar{x}$ (so $\bar{\bar{x}} = x$) and retraction $i\colon X \to V(X)$, where $V(X)$ is the fixed point set of $x \to \bar{x}$. Thus $i(i(x)) = i(x)$. One defines $\tau(x) =: i(\bar{x})$. Intuitively, $V(X)$ is the set of vertices, $X - V(X) =: E(X)$ is the set of edges, i is the initial vertex and τ is the terminal vertex.

A morphism $f\colon X \to X'$ of graphs is a function such that $f(\bar{x}) = \overline{f(x)}$, and $i(f(x)) = f(i(x))$, $x \in X$. The category of graphs has a final object, fibre products and push outs. In addition, there is a geometrical realization functor BX which assigns to X a 1-dimensional CW complex. This permits us to use the geometrical language of maximal trees, fundamental group and homotopy equivalence, although the notions have purely combinatorial definitions in the category of graphs.

A morphism $f\colon X \to Y$ is called an *immersion* [**St1**] if $f_v\colon \mathrm{Star}_v(X) \to \mathrm{Star}_{f(v)}(Y)$ is injective for each $v \in V(X)$. Here $\mathrm{Star}_v(X) = \{e \in E(X) \mid ie = v\}$. Immersions preserve reduced paths and hence induce injective maps on π_1 [**St1**]. Of crucial importance in our work is the *degenerate set* Df of a morphism $f\colon X \to Y$; here $Df =: \{x \in X \mid f(x) \in V(Y)\}$.

If $v \in V(X)$ and $f\colon X \to Y$ is a morphism, denote by f_* the induced map $\pi_1(X, v) \to \pi_1(Y, f(v))$.

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