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PERIODIC SOLUTIONS OF NONLINEAR VIBRATING STRINGS AND DUALITY PRINCIPLES

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Introduction. Our lecture deals with the study of T-periodic solutions for the nonlinear vibrating string equation:

$$(1)\qquad \begin{aligned} &u_{tt} - u_{xx} + g(u) = f(x, t), && 0 < x < \pi,\ t \in \mathbf{R},\\ &u(x, t) = 0, && x = 0,\ x = \pi,\ t \in \mathbf{R},\\ &u(x, t + T) = u(x, t), && 0 < x < \pi,\ t \in \mathbf{R}. \end{aligned}$$

Here g denotes a continuous function on $\mathbf{R}$ such that $g(0) = 0$ and $f(x, t)$ is a given T-periodic function of t.

Problem (1) may be viewed as an *infinite-dimensional Hamiltonian system* (let us recall that H. Poincaré has abundantly investigated the question of periodic solutions for finite-dimensional Hamiltonian systems; see **[50]**). Indeed if we set $p = u$ and $q = u_t$, then (1) becomes

$$\frac{\partial}{\partial t}\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} H_q \\ -H_p \end{pmatrix} + \begin{pmatrix} 0 \\ f \end{pmatrix}$$

where the Hamiltonian H is defined on the space $H_0^1(0, \pi) \times L^2(0, \pi)$ by

$$H(p, q) = \frac{1}{2}\int_0^\pi (p_x)^2\,dx + \int_0^\pi G(p)\,dx + \frac{1}{2}\int_0^\pi q^2\,dx$$

and G denotes a primitive of g.

We shall be concerned with two distinct questions.

Question 1. Existence of *forced vibrations*; that is, given $f(x, t)$ find at least one solution of (1).

Question 2. Existence of *free vibrations* (or "breathers"); that is, assume $f \equiv 0$ and find at least one *nonzero* solution of (1).

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