# FIFTY YEARS OF HOMOTOPY THEORY

### BY GEORGE W. WHITEHEAD

The subject of homotopy theory may be said to have begun in 1930 with the discovery of the Hopf map. Since I began to work under Norman Steenrod as a graduate student at Chicago in 1939 and received my Ph.D. in 1941, I have been active in the field for all but the first ten years of its existence. Thus the present account of the development of the subject is based, to a large extent, on my own recollections.

I have divided my discussion into two parts, the first covering the period from 1930 to about 1960 and the second from 1960 to the present. Each part is accompanied by a diagram showing the connections among the results discussed, and one reason for the twofold division is the complication of the diagram that would result were we to attempt to merge the two eras into one.

The dating given in this paper reflects, not the publication dates of the papers involved, but, as nearly as I can determine them, the actual dates of discovery. In many cases, this is based on my own memory; this failing, I have used the date of the earliest announcement in print of the result (for example, as the abstract of a paper presented to the American Mathematical Society or as a note in the *Comptes Rendus* or the Proceedings of the National Academy). Failing these, I have used the date of submission of the paper, whenever available. Only in the last resort have I used the actual publication date.

I wish to thank my many friends who have made pertinent comments, and helped refresh my memory on a number of points. Particular thanks are due to Saunders Mac Lane, William S. Massey, and Franklin P. Peterson. I also wish to acknowledge that my exposition of the solution of the immersion conjecture was based on a seminar talk by Professor Peterson on the same subject.

## PART ONE

I shall begin by describing the state of the art in 1930. The apparatus of simplicial homology had been set up, and extended to more general spaces: on the one hand, *via* the singular theory (which was, to be sure, in a somewhat unsatisfactory state, owing to the fact that the groups of singular chains were not free); on the other, *via* the Vietoris theory (valid for compacta—the generalization to arbitrary spaces by Čech was not to appear until 1932). The Alexander and Poincaré duality theorems and the Hopf-Lefschetz fixed point