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*Propriétés spectrales des algèbres de Banach*, by Bernard Aupetit, Lecture Notes in Math., vol. 735, Springer-Verlag, Berlin and New York, 1979, xii + 192 pp., $13.80.

**0. Introduction.** From the beginning of Banach algebra theory in the fundamental papers of Gelfand and his collaborators in the 1940s, particularly the classic paper [**6**], the spectrum has played a key role both in the general theory and in its many applications. Recall that if $a$ belongs to the complex Banach algebra $A$ with identity, then the *spectrum*, $\mathrm{Sp}(a)$ or $\mathrm{Sp}_A(a)$, of $a$ is the set of complex numbers $\lambda$ for which $\lambda - a$ is not invertible in $A$. Recall also that $\mathrm{Sp}(a)$ is a compact nonvoid subset of complex numbers [**3**, **15**]. To avoid technicalities in this review, we will not discuss real Banach algebras or algebras without identity, though Aupetit and the standard treatises [**3**, **15**] sometimes do.

The spectrum is an important and natural concept in the standard examples of Banach algebras that occur in applications. For operators on a Banach space the spectrum is the usual operator spectrum, and the essential spectrum or Fredhölm spectrum is just the spectrum in the Calkin algebra. For commutative group algebras like $L^1(\mathbf{R})$ with an identity adjoined, the spectrum of a function is just the range of its Fourier transform union $\{0\}$.

For $C(X)$, the algebra of continuous functions on the compact Hausdorff space $X$, the spectrum of a function is its range. For an arbitrary commutative Banach algebra $A$ the *Gelfand transformation* [**6**] is a continuous algebra homomorphism $a \to \hat{a}$ from $A$ to an appropriate $C(X)$ and, moreover, $\mathrm{Sp}_A(a) = \hat{a}(X)$. Thus the Gelfand transformation shows that the behavior of the spectrum is particularly simple in commutative Banach algebras. One focus of recent research is to determine to what extent simple spectral behavior entails commutativity.

The two standard treatises on Banach algebras [**15**, **3**] were published in 1960 and 1973, respectively, and thus do not describe the enormous number of interesting results discovered in the last decade. Aupetit's book is a very comprehensive report on those recent results which are related to spectral theory, many of which are due to Aupetit himself. (For excellent reports on the most important other recent results, see [**2**, **5**, **17**].) From now on we will usually refer to Aupetit's book as [AUP].

Many of the recent advances in spectral theory come from the exploitation, largely by Aupetit, but also by Jaroslav Zemánek and his collaborators, of the subharmonicity of various functions of the spectrum. Explicitly, if we let $\rho(a) = \max(\mathrm{Sp}(a))$ be the *spectral radius* of $a$ and let $\delta(a)$ be the diameter of $\mathrm{Sp}(a)$, and if $f(\lambda)$ is any analytic function from a domain of complex numbers to a Banach algebra, then $\rho(f(\lambda))$, $\log \rho(f(\lambda))$, $\delta(f(\lambda))$, and $\log \delta(f(\lambda))$ are all subharmonic [AUP, pp. 9–11]. In applications, $f(\lambda)$ is usually an explicit