which impelled her to go beyond the confines of the specialist's work. In many respects her inner vision complements that of C. F. Gauss. But, where it takes arduous studies and a life time of commitment to discover C. F. Gauss's program behind the cool marble of his finished works, Emmy Noether communicated her ideas in her active years freely and convincingly to many people and through them to subsequent generations of scholars. It is safe to predict that many more (and perhaps even more inspired) tributes to her life and work are going to appear in the future.

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An introduction to variational inequalities and their applications, by David Kinderlehrer and Guido Stampacchia, Academic Press, New York, 1980, xiv + 313 pp., $35.00.

The theory of variational inequalities (= V.I.) was born in Italy in the early sixties. The "founding fathers" were G. Stampacchia and G. Fichera. Stampacchia was motivated by potential theory, while Fichera was motivated by mechanics (problems in elasticity with unilateral constraints; see III.2). Less than twenty years later the theory of V.I. has become a rich source of inspiration *both* in pure and applied mathematics. On the one hand, V.I. have stimulated new and deep results dealing with nonlinear partial differential equations. On the other hand, V.I. have been used in a *large variety* of questions in mechanics, physics, optimization and control, linear programming, engineering, etc...; today V.I. are considered as an indispensable tool in various sectors of applied mathematics.

I. What is a V.I.? V.I. appears in a natural way in the *calculus of variations* when a function is minimized over a convex set of constraints. In this case the classical Euler equation must be replaced by a set of inequalities. Let us consider first a very simple example.

I.1. *V.I. in finite-dimensional spaces*. Let F denote a C^1 real valued function on $\mathbf{R}^n$ and let $K \subset \mathbf{R}^n$ be a closed convex set. If there is some $u \in K$ such that

$$F(u) = \operatorname*{Min}_{v \in K} F(v) \tag{1}$$

then u satisfies

$$\begin{cases} u \in K, \\ (F'(u), v - u) \geqslant 0 \quad \text{for all } v \in K. \end{cases} \tag{2}$$

In general a solution of (2) is not a solution of (1), unless F is convex.

EXAMPLE. $F(v) = |v - a|^2$ ($a \in \mathbf{R}^n$), then (2) reduces to the well-known characterization of the projection of a on K.