BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 7, Number 3, November 1982

0273-0979/82/0000-0246/$03.25

# BOOK REVIEWS

*Fuzzy sets and systems: Theory and applications*, by Didier Dubois and Henri Prade, Mathematics in Science and Engineering, vol. 144, Academic Press, New York, 1980, xvii + 393 pp., $49.50.

This book effectively constitutes a detailed annotated bibliography in quasi-textbook style of the some thousand contributions deemed by Messrs. Dubois and Prade to belong to the area of fuzzy set theory and its applications or interactions in a wide spectrum of scientific disciplines. The individual with an existing research commitment in this area will find the book competently written and an invaluable time-saver in developing an awareness of existing literature in English, French or German although the emphasis reflects, heavily at times, the authors' favorite topics. The mathematician wishing to precipitate out the mathematical and conceptual core of fuzzy set theory will find it frustrating reading, however. As I see it, the difficulty lies with how the subject is defined. Fuzzy set theory is not well delineated mathematically. It is determined by a set of papers which either have the word 'fuzzy' in the title or are authored by someone who has written such a paper. I am somewhat horrified that the authors include a bibliography specifically entitled 'nonfuzzy literature'. The dangerous insularity of the field has been noted in **[1, 17]**.

It seems justifiable, then, to direct this review in large part to fuzzy set theory generally.

**1. What passes for a theory.** This section briefly overviews the main aspects of the 'mathematical core' of fuzzy set theory as represented by the first two chapters of "Part II: Mathematical Tools" (such unqualified references are to the book under review). For lack of major theorems and juicy open questions, I do not feel this material has yet coagulated into what mathematicians would call a theory.

By identifying subsets of a set with their characteristic functions, the Boolean algebra structure of the set of subsets of a set derives, via pointwise operations, from the Boolean algebra structure of the two-element set $2 = \{0, 1\}$, 0 for 'false' and 1 for 'true'. Fuzzy set theory generalizes $2^X$ to $[0, 1]^X$ where $[0, 1]$ is the unit interval. Elements of $[0, 1]^X$ are called *fuzzy sets* (*with universe* $X$). The rigorization of probability theory by Kolmogoroff developed from a frequency paradigm. As discussed in Chapter 1 of Part IV, fuzzy sets have a more 'subjective' paradigm. (As a mathematician, I found this explicit foundational link to human psychology disquieting; perhaps the authors did too, since the placement of this discussion is far from the beginning of the book.) As a result, there is little intuitive basis to decide which operations $[0, 1]^n \to [0, 1]$ should generalize the Boolean operations $2^n \to 2$. The authors avoid this problem by compiling a voluminous undifferentiated collection of ad-hoc operations from the literature.