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NEW DEFECT RELATIONS FOR MEROMORPHIC FUNCTIONS ON $\mathbf{C}^n$

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For meromorphic functions f and φ on the complex line $\mathbf{C}^1$, one considers the *counting function* $N(r,\varphi,f) = \int_1^r n(t,\varphi,f)t^{-1}\,dt$, where $n(t,\varphi,f)$ denotes the number of solutions of the equation $f = \varphi$ (counting multiplicities) on the disk $\{|z| \leq t\}$. If $T(r,\varphi) = o(T(r,f))$, one defines the *defect* $\delta(\varphi,f) = \liminf[1 - N(r,\varphi,f)/T(r,f)]$ and observes that $0 \leq \delta(\varphi,f) \leq 1$ as in the case where φ =constant. In 1929, R. Nevanlinna [**4**, p. 77] asked if the *defect relation*

$$(*) \qquad \sum_{j=1}^{q} \delta(\varphi_j, f) \leq 2$$

is valid for distinct meromorphic functions φ_j with $T(r,\varphi_j) = o(T(r,f))$. The case where the φ_j are constant is Nevanlinna's fundamental defect relation [**4**]. (If $q = 3$, then $(*)$ follows immediately from the Nevanlinna defect relation.) In 1939, J. Dufresnoy [**3**] showed that $\sum \delta(\varphi_j,f) \leq d+2$ if f is transcendental and the φ_j are distinct polynomials of degree$\leq d$. In 1964, C.-T. Chuang [**2**] gave a general Second Main Theorem which yields $(*)$ for the case where f is holomorphic (or more generally when $\delta(\infty,f) = 1$) and which generalizes the defect relation of Dufresnoy [**3**]. However, this question of Nevanlinna remains unanswered today even for polynomial φ_j, despite Nevanlinna's assertion [**4**, p. 77] that $(*)$ "follows easily" for this case. If f is a meromorphic function on $\mathbf{C}^n$, then a special case of a theorem of W. Stoll [**7**] (see also Vitter [**8**]) yields $(*)$ for constant φ_j as in the classical Nevanlinna theory. (In fact, the results of Chuang [**2**] generalize easily to $\mathbf{C}^n$.) In this note we announce a new defect relation of the form $(*)$ for meromorphic functions on $\mathbf{C}^n$, $n \geq 2$.

If f and φ are distinct meromorphic functions on $\mathbf{C}^n$, we let $D(\varphi,f)$ denote the divisor on $\mathbf{C}^n$ given by the solution (with multiplicities) to the equation $f = \varphi$. We write $N(r,\varphi,f) = N(r,D(\varphi,f))$, where $N(r,D)$ denotes the counting function for D as given in [**1** or **7**]. We easily obtain the *First Main Theorem*,

$$N(r,\varphi,f) + m(r,\varphi,f) = T(r,f) + T(r,\varphi) + c,$$

where the *proximity term* $m(r,\varphi,f) \geq 0$. Our main result is the following

SECOND MAIN THEOREM. *Let f, $\varphi_1,\ldots,\varphi_q$ be distinct meromorphic functions on $\mathbf{C}^n$ ($q \geq n-1$) such that*

(i) $\operatorname{rank}(\varphi_1,\ldots,\varphi_q) = n-1$,

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