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A NONLINEAR PARTIAL DIFFERENTIAL EQUATION AND THE UNCONDITIONAL CONSTANT OF THE HAAR SYSTEM IN L^p

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1. Introduction. Our aim here is to identify the best constant in an inequality (see Theorem 1) that has proved useful in the study of singular integrals, stochastic integrals, the structure of Banach spaces, and in several other areas of study. Our work yields the unconditional constant of the Haar system in $L^p(0,1)$ and rests partly on solving the nonlinear partial differential equation

$$(1) \qquad (p-1)[yF_y - xF_x]F_{yy} - [(p-1)F_y - xF_{xy}]^2 + x^2 F_{xx}F_{yy} = 0$$

for F nonconstant and satisfying other conditions on a suitable domain of $\mathbf{R}^2$.

We assume throughout that $1 < p < \infty$ and write L^p for the real Lebesgue space $L^p(0,1)$. The *unconditional constant* $K_p(e)$ of a sequence $e = (e_1, e_2, \ldots)$ in L^p is the least $K \in [1, +\infty]$ with the property that if n is a positive integer and $a_1, \ldots, a_n$ are real numbers such that $\|\sum_{k=1}^n a_k e_k\|_p = 1$, then

$$\left\| \sum_{k=1}^{n} \epsilon_k a_k e_k \right\|_p \leq K$$

for all choices of signs $\epsilon_k \in \{-1, 1\}$. The sequence e is a *basis* of L^p if, for every $f \in L^p$, there is a unique sequence a such that $\|f - \sum_{k=1}^n a_k e_k\|_p \to 0$ as $n \to \infty$. A sequence $d = (d_1, d_2, \ldots)$ in L^p is a *martingale difference sequence* if d_{n+1} is orthogonal to $\varphi(d_1, \ldots, d_n)$ for all bounded continuous functions $\varphi\colon \mathbf{R}^n \to \mathbf{R}$ and all $n \geq 1$.

The Haar system $h = (h_1, h_2, \ldots)$ is both a basis of L^p (Schauder [**11**]) and a martingale difference sequence: h_{n+1} is supported by a set on which $\varphi(h_1, \ldots, h_n)$ is constant. By an inequality of Paley (see [**10** and **6**]), its unconditional constant $K_p(h)$ is finite. More generally [**2**], $\sup_d K_p(d)$ is finite where the supremum is taken over all martingale difference sequences d in L^p. In fact, if d is a martingale difference sequence and e is a basis of L^p, then

$$(2) \qquad K_p(d) \leq K_p(h) \leq K_p(e).$$

The right-hand side of (2) is due to Olevskiĭ[**8, 9**] and the left-hand side to Maurey [**7**]. Lindenstrauss and Pełczyński [**5**] have an alternative approach to

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