

ELEMENTARY METHODS IN THE STUDY OF THE DISTRIBUTION OF PRIME NUMBERS¹

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1. Historical survey. The question of how primes are distributed among all natural numbers has long been a source of fascination. The oldest known result in this area is the classical theorem that there are an infinite number of primes. The proof of this theorem recorded by Euclid [Euc] has stood as one of the most beautiful in mathematics. No more detailed information about the distribution of primes was known in ancient times, for their apparently random occurrence frustrated all attempts at obtaining simple precise formulas.

We take up our account at the end of the 18th century, when the "right question" about the distribution of primes was asked and a conjectured answer offered by A. M. Legendre [Leg] and by C. F. Gauss [Gau]. They recast the prime distribution question in a statistical form: About how many of the first N positive integers are primes? Examination of tables of prime numbers led them to conjecture that the answer was, in some sense, $N/\log N$.

If we let $\pi(x)$ denote the number of primes in the interval $[1, x]$, the assertion of Gauss and Legendre can be expressed concisely as the celebrated Prime Number Theorem (P.N.T.).

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