

SCHRÖDINGER SEMIGROUPS

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ABSTRACT. Let $H = -\frac{1}{2}\Delta + V$ be a general Schrödinger operator on R^n ($n \geq 1$), where Δ is the Laplace differential operator and V is a potential function on which we assume minimal hypotheses of growth and regularity, and in particular allow V which are unbounded below. We give a general survey of the properties of e^{-itH} , $t > 0$, and related mappings given in terms of solutions of initial value problems for the differential equation $du/dt + Hu = 0$. Among the subjects treated are L^p -properties of these maps, existence of continuous integral kernels for them, and regularity properties of eigenfunctions, including Harnack's inequality.

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