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BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 7, Number 2, September 1982
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0273-0979/82/0000-0121/\$01.50

Operator inequalities, by Johann Schröder, Mathematics in Science and Engineering, vol. 147, Academic Press, New York, 1980, xvi + 367 pp., \$39.50.

There exists an extensive literature on the theory of differential inequalities relative to initial value problems in finite and infinite dimensional spaces, including random differential inequalities [2, 3, 5, 6–8, 9]. This theory is also known as the theory of comparison principle. The corresponding theory of differential inequalities related to boundary value problems of ordinary and partial differential equations has also developed substantially [1, 4, 5, 9]. The treatment of this general theory of differential inequalities is not for its own sake. The essential unity is achieved by the wealth of its applications to various qualitative and quantitative problems of a variety of dynamical systems. This theory can be applied employing as a candidate a suitable norm or more generally a Lyapunov-like function, to provide an effective mechanism for investigating various problems. It is therefore natural to expect the development of an abstract theory so as to bring out the unifying theme of various theories of inequalities. The present book is an attempt in this direction.

As the title suggests, this book is concerned with inequalities that are described by operators which may be matrices, differential operators, or integral operators. As an example the inverse-positive linear operators M , may be described by the property that $Mu \geq 0$ implies $u \geq 0$. These are operators M which have a positive inverse M^{-1} . For an inverse-positive operator M one can derive estimates of $M^{-1}r$ from properties of r without knowing the inverse M^{-1} explicitly. This property can be used to derive a priori estimates for solutions of equations $Mu = r$. There are important applications as well. For example, if M is inverse-positive, an equation $Mu = Nu$ with a nonlinear operator N may be transformed into a fixed-point equation $u = M^{-1}Nu$, to which then a monotone iteration method or other methods may be applied, if N has suitable properties. Moreover, for inverse-positive M the eigenvalue