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ON THE NORMALIZER OF CERTAIN SUBALGEBRAS OF GROUP-MEASURE FACTORS

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ABSTRACT. We study operator algebras constructed from ergodic actions of discrete groups on compact Lebesgue spaces. We show that under appropriate conditions, the normalizer of a subalgebra corresponding to a quotient action depends on the relative elementary spectrum of the action over the quotient action. Using these results and methods of groupoid cohomology we prove the existence of an uncountable family of Cartan subalgebras of the hyperfinite II_1 factor, no two of which are inner conjugate.

1. Introductory remarks. Let the countable discrete group G act freely and ergodically on the compact Lebesgue space (X, μ) so as to leave the finite measure μ quasi-invariant. By the von Neumann-Murray group-measure construction we then may form the factor $F(X, G)$. Let (Y, ν, G) be a *quotient action* of (X, μ, G); by this we mean that there exists a surjective Borel map $\varphi: X \longrightarrow Y$ satisfying $\varphi^*(\mu) = \nu$ and $\varphi(xg) = \varphi(x)g$ (μ a.e.) for every g in G; we also call (X, μ, G) an extension of (Y, ν, G). Throughout the following we will assume that in the decomposition of μ over the fibers of φ as $\mu = \int \mu_y d\nu$, $\mu_{yg} = g^*(\mu_y)$. This implies that there exists a G-invariant conditional expectation of $L^\infty(X)$ onto $L^\infty(Y)$. The map φ provides a homomorphism of the ergodic measure groupoid $X \times G$, onto the groupoid $Y \times G$, and one thus obtains an injection of $F(Y, G)$ into $F(X, G)$, which we shall denote by φ^*. Since G acts ergodically on (X, μ) it follows that G acts ergodically on (Y, ν), and if G acts freely on (Y, ν) as well, $F(Y, G)$ injects naturally as a subfactor of $F(X, G)$.

THEOREM 1. *Let (Y, ν, G) be a free quotient action of (X, μ, G) where X and Y are compact Lebesgue spaces and μ* (hence ν) *is finite and G-invariant, and where the countable discrete group G acts freely and ergodically on (X, μ). Then there is a canonical correspondence between the intermediate subalgebras of $F(Y, G)$ and $F(Y, G)$ and the quotient actions (Z, G) of (X, G) which are extension actions of (Y, G).*

REMARK 1. Since (Z, G) will inherit ergodicity from (X, G) and freeness from (Y, G), it follows immediately from the theorem that any intermediate subalgebra, which will be of the form $F(Z, G)$ for some (Z, G), will be a subfactor.

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