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RESEARCH ANNOUNCEMENTS

NEW EXAMPLES OF MINIMAL IMBEDDINGS OF S^{n-1} INTO $S^n(1)$—THE SPHERICAL BERNSTEIN PROBLEM FOR $n = 4, 5, 6$

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The classical Bernstein theorem proves that an entire minimal graph in $\mathbf{R}^3$ is necessarily a plane. Analytically speaking, an entire minimal graph in $\mathbf{R}^{n+1}$ is given by an entire solution, $u(x^1, \ldots, x^n) \in C^2(\mathbf{R}^n)$, of the following minimal equation

$$\sum_{i=1}^{n} D_i \frac{D_i u}{\sqrt{1 + |Du|^2}} = 0.$$

The Bernstein problem asks whether an *entire* solution of the above equation is necessarily a *linear* function. The above problem was proved to be affirmative in the cases $n = 3$ by De Giorgi [6], $n = 4$ by Almgren [1] and $n \leqslant 7$ by Simons [9]. In the remaining cases of $n \geqslant 8$, it was settled to be negative by Bombieri, De Giorgi and Guisti in 1969 [2]. The study of Bernstein problem is closely related to that of minimal cones, singularities of minimal hypersurfaces and closed minimal hypersurfaces of the diffeomorphic type of $\mathbf{R}^{n-1}$ in E^n and that of the diffeomorphic type of S^{n-1} in $S^n(1)$. They are clearly simple testing problems of fundamental theoretical importance. For example, the following so-called spherical Bernstein problem was proposed by S. S. Chern in 1969 [4] and again in his address to International Congress of Mathematicians at Nice, 1970 [5] as an outstanding problem in differential geometry.

SPHERICAL BERNSTEIN PROBLEM. Let the $(n-1)$-sphere be *imbedded* as a minimal hypersurface in $S^n(1)$. Is it (necessarily) an equator?

The beginning case of $n = 3$ was known even before the above problem was proposed, namely, a theorem of Almgren [1] and Calabi [3]. So far, no progress has been made in the positive direction. We announce here the construction of infinitely many distinct new examples of *minimal imbeddings* of S^{n-1} into $S^n(1)$ for the cases $n = 4$, 5 and 6. Our construction makes use of the framework of

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