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ON THE COHOMOLOGY OF CHEVALLEY GROUPS[1]

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Let G be a simple, simply connected algebraic group defined and split over the prime field F_p, $p \geqslant 3$. Let V be a finite dimensional rational G-module. As shown in [1], for d suitably large, the Eilenberg-Mac Lane cohomology groups $H^*(G(F_p d), V)$ achieve a stable value $H^*_{\text{gen}}(G, V)$, the so-called generic cohomology of V. This generic cohomology can in turn be determined from the rational cohomology groups $H^*_{\text{rat}}(G, V^{(r)})$, $r \geqslant 1$, where $V^{(r)}$ stands for the module V with the action of G twisted by the rth power of the Frobenius morphism $\sigma\colon G \longrightarrow G$. In this paper we announce an explicit determination of the cohomology groups $H^*_{\text{rat}}(G, V^{(r)})$, and hence of the generic cohomology groups $H^*_{\text{gen}}(G, V)$, in a range of cohomology degrees (restricted by the prime p) for an arbitrary irreducible rational module V whose high weight lies in the bottom p-alcove. In particular, we obtain stability in $H^n_{\text{gen}}(G, V)$ for large p, answering a question raised by Scott [6]. Our methods involve a determination in a range of degrees of the cohomology of the restricted enveloping algebra of the Lie algebra of G.

The general Theorem 4 below is motivated by the "experimental evidence" provided by Theorem 1. This result concerning the general linear group is particularly strong in terms of its range of degrees, its applicability to small fields, and its description of the k-algebra structure. Here k is an algebraically closed field of characteristic p.

THEOREM 1. *The following k-algebras are isomorphic in degrees less than* $\min(2p - 1, d(2p - 3) - 2)$, *where* $q = p^d$:

$$H^*(GL_n(F_q), M_n); \qquad H^*_{\text{rat}}(GL_n, M_n^{(r)}), \quad r \geqslant 1; \qquad k[t]/(t^n), \ \deg(t) = 2.$$

In Theorem 1, M_n is viewed as the adjoint representation of the algebraic group GL_n over k. We emphasize that $H^*(GL_n(F_q), M_n)$ and $H^*_{\text{rat}}(GL_n, M_n^{(r)})$ have natural k-algebra structures because GL_n acts as (associative) k-algebra automorphisms on M_n. These multiplicative structures are analyzed using external comultiplications in cohomology induced by external tensor product maps $GL_m \times GL_n \longrightarrow GL_{mn}$.

The key step in the proof of Theorem 1 is the identification of

$$H^*(B_n(F_q), M_n)$$

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