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# TORSION FREE SUBGROUPS OF FUCHSIAN GROUPS AND TESSELLATIONS OF SURFACES[1]

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It has been known for many years that a finitely generated fuchsian group $G$ i.e. a finitely generated discrete subgroup of orientation-preserving isometries of the hyperbolic plane contains a torsion free subgroup of finite index. The known proofs are by representations in the symmetric groups cf. Fox [3] or by the method of congruence subgroups cf. Mennicke [4]. The latter method extends to all finitely generated matrix groups cf. Selberg [5]. There is no information about the possible indices of subgroups in these proofs. Here we announce the precise determination of the possible indices of torsion free subgroups of finite index in terms of the torsion in $G$. Using the connection between fuchsian groups and uniformization of Riemann surfaces the results may be interpreted as a step in determining a class of intermediate uniformizations, or looked in a different way, a step towards a topological classification of holomorphic maps between Riemann surfaces of finite type. Contained herein are some results of a naive geometric interest. Namely they imply the existence of certain interesting tessellations of surfaces which are natural generalizations of the tessellations of the sphere determined by Platonic solids. We remark that we completely leave aside the questions of normality of subgroups. Determining the indices of *normal* torsion-free subgroups of finite index in fuchsian groups appears to involve deeper number theoretic considerations which are probably yet to be understood.

To formulate our main result let $G$ have a standard presentation with generators $a_1, b_1, \ldots, a_g, b_g, x_1, \ldots, x_r, y_1, \ldots, y_s$ and relations $x_1^{m_1} = \cdots = x_r^{m_r} = 1$ and $a_1 b_1 a_1^{-1} b_1^{-1} \cdots a_g b_g a_g^{-1} b_g^{-1} x_1 \cdots x_r y_1 \cdots y_s = 1$. Let $l = \mathrm{LCM}\{m_1, \ldots, m_r\}$, and let $l_{(2)}$ denote the 2-primary part of $l$. We say that $G$ has *odd type* if $s = 0$, $l_{(2)} > 1$, and the number of $m_i$'s such that $l_{(2)} | m_i$ is odd. Otherwise $G$ has *even type.*

THEOREM 1. *The infinite fuchsian group $G$ contains a torsion free subgroup of index $k$ if and only if $k \equiv 0$ modulo $2^\epsilon l$, where $\epsilon = 0$ if $G$ has even type and $\epsilon = 1$ if $G$ has odd type.*

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