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A GENERAL REGULARITY THEOREM FOR SEMILINEAR HYPERBOLIC WAVES IN ONE SPACE DIMENSION

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We study the propagation of singularities for semilinear strictly hyperbolic systems in one space dimension

(1) $$D_i u_i = f_i(x, t, u), \quad u_i(x, 0) = u_i^{(0)}(x),$$

$i = 1, \ldots, m$, where the vector fields D_i and the functions f_i are C^∞. In [1] we presented a simple example which shows that the propagation of singularities for (1), is, in general, not the same as for the special case where the f_i are linear in u. In [2] we used elementary methods to study this problem when the initial data are piecewise smooth. We report here on results which permit general initial data $u^{(0)} \in H^s_{\text{loc}}$, $s > ½$.

The propagation of singularities for (1) is governed by two general principles:

TREE LAW. When two or more singularity bearing characteristics intersect, the point of intersection becomes a source of singularities travelling, in general, in all forward characteristic directions from the point.

SUM LAW. When H^s singularities collide with H^τ singularities, the new singularities produced will have strength $H^{s+\tau}$ (this must be interpreted in a suitable microlocal sense).

Let $S^{(0)}$ denote the union of forward characteristics from the singular support of the initial data. Let $S^{(1)}$ denote the union of forward characteristics from points of intersection in $S^{(0)}$. Continue defining $S^{(l)}$ in this way and set $S =$ closure $\bigcup^\infty S^{(l)}$. The first general principle indicates that, in general, S will be the singular support of u. The second general principle predicts the regularity of u at various points in S.

EXAMPLE 1. Let $m = 3$, $D_1 = \partial/\partial t, + \partial/\partial x$, $D_2 = \partial/\partial t - \partial/\partial x$, $D_3 = \partial/\partial t$ and suppose that $u^{(0)}$ is H^s on $[a, b]$ and C^∞ outside $[a, b]$. Then the two general principles predict the regularity depicted in Figure 1. If the f_i are linear, then u will be C^∞ in the regions labelled $H^{2s}, H^{3s}, \ldots$.

EXAMPLE 2. Suppose that $m = 4$ and that there are two rightward moving and two leftward moving characteristics. Suppose that the data is C^∞ except at two points x_1 and x_2.

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