BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 6, Number 3, May 1982

EUCLIDEAN, HYPERBOLIC AND SPHERICAL BLOCH CONSTANTS

BY C. DAVID MINDA[1]

TO PROFESSOR S. E. WARSCHAWSKI ON HIS 78TH *BIRTHDAY*

We begin with a brief survey of some of the known results dealing with Bloch constants. Bloch's theorem [3] asserts that there is a constant $\mathcal{B}_1 > 0$ such that if f is holomorphic in the open unit disk $\mathbf{B}$ and normalized by $|f'(0)| \geqslant 1$, then the Riemann surface of f, viewed as spread over the complex plane $\mathbf{C}$, contains an unramified disk of radius at least $\mathcal{B}_1$. Pommerenke [11] introduced the locally schlicht Bloch constant $\mathcal{B}_\infty > \mathcal{B}_1$ which has the same property relative to the family of normalized locally schlicht holomorphic functions defined on $\mathbf{B}$. He showed that $\mathcal{B}_\infty \leqslant \mathfrak{L}$, where $\mathfrak{L}$ denotes the Landau constant [7]. The precise values of these constants are not known; however, the following bounds are known.

$$.433 < \frac{\sqrt{3}}{4} < \mathcal{B}_1 \leqslant \frac{1}{\sqrt{1+\sqrt{3}}} \frac{\Gamma(1/3)\Gamma(11/12)}{\Gamma(1/4)} < .4719,$$

$$\frac{1}{2} < \mathcal{B}_\infty \leqslant \mathfrak{L} \leqslant \frac{\Gamma(1/3)\Gamma(5/6)}{\Gamma(1/6)} < .5433.$$

The lower bounds for $\mathcal{B}_1$ and $\mathcal{B}_\infty$, but without strict inequality, are the work of Ahlfors [1]. The strict inequalities were established by Heins [6] and Pommerenke [11], respectively. The upper bound for the Bloch constant comes from an example of Ahlfors and Grunsky [2] and is conjectured to be sharp. The upper bound for the Landau constant that is frequently cited is .544; this is due to an unpublished example of R. M. Robinson that is mentioned in [1]. In [8] we present an explicit example which yields the upper bound for the Landau constant that is given above. It is analogous to the Ahlfors-Grunsky example and it is plausible that it should give the actual value of the Landau constant. It is probably the example of Robinson. In fact, in [8] we exhibit a unified approach to obtaining upper and lower bounds for these and other new Bloch constants. As special cases of our results we obtain all of the previously mentioned bounds.

Received by the editors September 11, 1981.

1980 *Mathematics Subject Classification.* Primary 30C25, 30C80.

[1]This research was completed while the author was visiting the University of California, San Diego during 1980–81.