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# RESEARCH ANNOUNCEMENTS

## ON $K_3$ AND $K_4$ OF THE INTEGERS MOD $n$

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Quillen [7] defines an algebraic $K$-functor from the category of associative rings to that of positively graded abelian groups, with $K_i(R) = \Pi_i(\mathrm{BGLR}^+)$ for $i \geq 1$. $K_1$ and $K_2$ correspond respectively to the 'classical' Bass and Milnor definitions. The $K$-images of finite fields and their algebraic closures were computed by Quillen in [8]. Since then, there has been only a handful of complete calculations of any of the higher $K$-groups ($K_i$ for $i > 2$). Lee and Szczarba [4] showed that the Karoubi subgroup $Z/48$ of $K_3(Z)$ was the full group. Evens and Friedlander [3] computed $K_i(Z/p^2)$ and $K_i(\mathrm{F}_p[t]/(t^2))$ for $i < 5$ and prime $p$ greater than 3. Snaith in [1] and, with Lluis, in [5], fully determined $K_3(\mathrm{F}_{p^m}[t]/(t^2))$ for $m \geq 1$ and prime $p$ other than 3.

This note summarizes computations of the groups $K_3(Z/n)$, and $K_4(Z/p^k)$ for $k > 1$ and prime $p > 3$. These complete the recent partial results on $K_3(Z/4)$ by Snaith and on $K_3(Z/9)$ by Lluis, and extend the work of Evens and Friedlander. The theorem stated below is consistent with the Karoubi conjecture that for odd primes, $\mathrm{BGL}Z/p^{k+}$ is the homotopy fibre of the difference of Adams operations, $\Psi^{p^k} - \Psi^{p^{k-1}}$. However, Priddy [6] has disproved the conjecture in the cases $p > 3$ and $k = 2$.

I am most grateful to Victor Snaith for his supervision of the thesis in which these results originally appeared. Details of the proofs can also be found in [1].

THEOREM. *Take* $k > 1$ *and* $0 < i \leq 2$.

(a) $K_{2i-1}(Z/2^k) = Z/2^i \oplus Z/2^{i(k-2)} \oplus Z/(2^i - 1)$. $K_{2i-1}(Z/p^k) = Z/p^{i(k-1)} \oplus Z/(p^i - 1)$ *if $p$ is an odd prime. For all primes, the map*

$$K_{2i-1}(Z/p^{k+1}) \longrightarrow K_{2i-1}(Z/p^k)$$

*induced by reduction is the obvious surjection.*

(b) *For prime* $p > 3$, $K_{2i}(Z/p^k) = 0$. $K_2(Z/3^k) = 0$. $K_2(Z/2^k) = Z/2$.

$K_1$ is due to Bass, $K_2$ to Milnor, Dennis Stein.

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