ERGODIC THEORY, GROUP REPRESENTATIONS, AND RIGIDITY

BY ROBERT J. ZIMMER¹

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These notes represent a mildly expanded version of lectures delivered at the C.I.M.E. summer session on harmonic analysis and group representations in Cortona, Italy, June—July 1980. The author would like to express his thanks and appreciation to the organizers of the conference, Michael Cowling, Sandro Figà-Talamanca, and Massimo Picardello, for inviting him to deliver these lectures and for their most warm and generous hospitality during his stay in Italy. We would also like to thank the other participants of the conference for their interest in these lectures. Finally, we would like to thank Terese S. Zimmer for (among innumerable other things that we need not go into here) helping with the translation of [29].

ADDED IN PROOF. Complete and detailed proofs of all the results discussed in these lectures (except those in section 4), as well as further developments, will appear in a forthcoming monograph of the author.

1. Basic notions. In these lectures we discuss some topics concerning the relationship of ergodic theory, representation theory, and the structure of Lie groups and their discrete subgroups.

In studying the representation theory of groups, the assumption of compactness on the group essentially allows one to reduce to a finite dimensional situation, in which case one often can obtain complete information. For noncompact groups, of course, no such reduction is possible and the situation is much more complex. When studying general actions of groups, a somewhat

© 1982 American Mathematical Society 0273-0979/81/0000-0334/\$09.50

Received by the editors November 15, 1981.

¹⁹⁸⁰ Mathematics Subject Classification. Primary 22D10, 22D40, 22E40, 28D15, 53C35, 57R30, 57S20.

¹Partially supported by a Sloan Foundation Fellowship and NSF Grant MCS 79-05036.