

LORENTZIAN FORMS FOR THE LEECH LATTICE

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ABSTRACT. Using recent results about holes in the Leech lattice we establish some Lorentzian constructions for that lattice.

Some years ago the first author and R. T. Curtis showed by a detailed (and unpublished) calculation that the set of points of the Lorentzian integer lattice $\mathbb{Z}^{24,1}$ which are perpendicular to the vector

$$t = (3, 5, 7, \dots, 45, 47, 51 \mid 145)$$

is a copy of the Leech lattice. Our recent work [1, 3] on holes in the Leech lattice enables us to give a short proof of this fact. We work instead in the hyperplane of vectors $v \in \mathbb{Z}^{24,1}$ with $v \cdot t = -2$, and observe that this contains all the points mentioned in Figure 1. Two points in the figure are joined by an edge if they are distant $\sqrt{6}$, all other pairs being distant $\sqrt{4}$ apart. Since Figure 1 is a copy of the D_{24} hole diagram (see [3]) this proves the result.

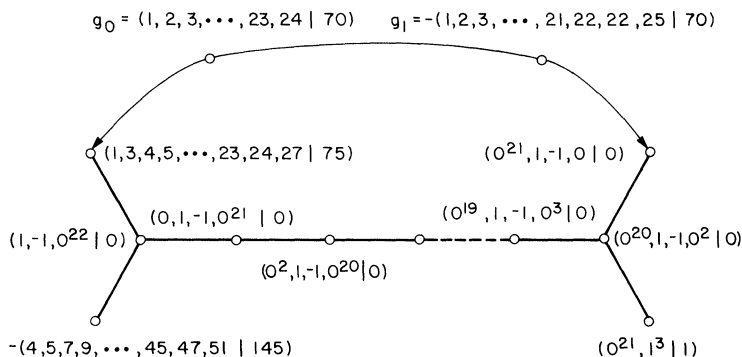


FIGURE 1. Hole diagram of type D_{24} for Leech lattice.

Seidel ([7]; see also Coxeter [4, p. 419] and Neumaier [5]) has recently remarked that elegant coordinates for the lattice E_8 may be obtained by considering the points of $\mathbb{Z}^{9,1}$ orthogonal to the isotropic vector

$$w = (1, 1, 1, 1, 1, 1, 1, 1, 1 \mid 3)$$

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