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GRAEME SEGAL'S BURNSIDE RING CONJECTURE

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1. Introduction. My theme will be that algebraic topology still offers problems which reveal the present state of our art as inadequate. Such problems make us feel that there could be something good going on; but if there is, we have yet to understand it.

I shall devote §2 to explaining Segal's original conjecture. In §3–§6 I shall review what is proved about it so far. Broadly, for those few groups we have been able to handle the conjecture is found to be true, and there is no group for which the conjecture is known to be false. In §7–§9 I shall explain further conjectures related to the original one; the fact that these also are not yet disproved contributes to the impression that there could be something good going on. In §10 I shall comment briefly on our chances of going further.

2. Statement of the conjecture. In this section I shall explain Segal's original conjecture. I must begin by explaining cohomotopy.

Let X, Y be a good pair of spaces, for example, a finite-dimensional CW-pair. Then we have

$$\pi^n(X, Y) = \operatorname*{Lim}_{m \to \infty} [S^m X/Y, S^{m+n}].$$

Here maps and homotopies preserve the base-point; and if $Y = \phi$, I use Atiyah's convention that X/ϕ means X with a disjoint base-point. We can rewrite the right-hand side as

$$\operatorname*{Lim}_{m \to \infty} [X/Y, \Omega^m S^{m+n}]$$

or as

$$\left[X/Y, \operatorname*{Lim}_{m \to \infty} \Omega^m S^{m+n}\right].$$

The last expression gives us a definition of $\pi^n(X, Y)$ valid whether the pair X, Y is finite dimensional or not.

Cohomotopy is a generalised cohomology theory, namely the one corresponding to the sphere spectrum; and its coefficient groups are the stable homotopy groups of spheres. So it is like stable homotopy; if you could compute it, it would give you a lot of information, but unfortunately it is hard to compute.

However, there is one case in which we have a conjecture, and it is due to Graeme Segal. He asks for the analogue of a well-known theorem of Atiyah [3].

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