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POINCARÉ AND ALGEBRAIC GEOMETRY

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ABSTRACT. A few of the contributions of Poincaré to algebraic geometry are described, with emphasis on his late work on normal functions. A very brief description of some recent works is also given.

Although the subject of algebraic geometry was not one of Poincaré's major preoccupations, his work in the field showed characteristic insight and brilliance and certainly has had a lasting effect. I shall briefly describe his major contributions, with special emphasis on the two papers on normal functions which constituted Poincaré's last published mathematical work.

In general I have tried to follow standard current notations. Unless mentioned to the contrary, homology and cohomology are with **Q** coefficients.

At the end there is a list of some of Poincaré's major papers in algebraic geometry together with the other bibliographical items referred to in the text.

As will be clear from the discussion below, the decision to emphasize Poincaré's work on normal functions reflects my own particular interest.

(a) Let me begin by recalling the period during which Poincaré worked. This was a particularly active time in algebraic geometry, especially for the study of algebraic surfaces. In the preceding thirty years, beginning with Riemann's thesis the theory of algebraic functions of one variable, or as we now know it, algebraic curves, had been developed by Riemann, Max Noether, and many others to the point where—aside from moduli questions—the theory had assumed much the form that we find it today. Beginning in the 1890's Poincaré's colleague, E. Picard, had begun his monumental work on transcendental algebraic geometry in higher dimensions, especially on surfaces, that among other things led to the Picard-Fuchs equation and Picard-Lefschetz transformation, the algebraic de Rham theorem for smoothly compactifiable affine varieties and subsequent theory of single and double integrals of the second kind on surfaces, and the Picard variety and Picard number. Meanwhile, in Italy the birational theory of algebraic surfaces was well underway in the work of Castelnuovo, Enriques, Severi (at a slightly later time), and the other members of the Italian school.

It is clear that Poincaré was well abreast of these developments, and it seems to me that his own work in algebraic geometry was frequently in

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