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RESEARCH ANNOUNCEMENTS

SPECTRAL PROPERTIES OF SOME NONSELFADJOINT OPERATORS[1]

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ABSTRACT. Let A be a compact linear operator on a Hilbert space H, $s_n(A) = \{\lambda_n(A^*A)\}^{1/2}$, Q be a compact linear operator, $I + Q$ be invertible, $B = A(I + Q)$. We prove that $s_n(B)s_n^{-1}(A) \to 1$ as $n \to \infty$. If $|Qf| \leqslant c|Af|^a|f|^{1-a}$, $a > 0$, $c > 0$, $f \in H$ and $s_n(A) = c_1 n^{-r}\{1 + O(n^{-q})\}r$, $q > 0$, then $s_n(B) = s_n(A)\{1 + O(n^{-\gamma})\}$, where $\gamma = \min\{q, ra(1 + ra)^{-1}\}$. This estimate is close to sharp. We also give conditions sufficient for the root system of B to form a Riesz basis with brackets of H. Applications to elliptic boundary value problems are given.

1. Notations, definitions. Let H be a separable Hilbert space, A and Q be compact linear operators on H, $B = A(I + Q)$, $\lambda_n(A)$ be the eigenvalues of A, $s_n(A) = \lambda_n\{(A^*A)^{1/2}\} = \{\lambda_n(A^*A)\}^{1/2}$ be the s-values of A (singular values of A), c be various positive constants, $\mathbf{R}^d$ be the Euclidean d-dimensional space, $D \subset \mathbf{R}^d$ be a bounded domain with a smooth boundary, L be a positive definite in $L^2(D)$ elliptic operator of order l and M be a nonselfadjoint differential operator of order $m < l$. We define $s_n(L) = \{s_n(L^{-1})\}^{-1}$. Let $A\phi = \lambda\phi$, $\phi \neq 0$. With the pair (λ, ϕ) one associates the Jordan chain defined as follows: consider $(*)$ $A\phi^{(1)} - \lambda\phi^{(1)} = \phi$. If this equation is not solvable then one says that there are no root vectors associated with the pair (λ, ϕ). If $(*)$ is solvable then consider the equations $(**)$ $A\phi^{(j)} - \lambda\phi^{(j)} = \phi^{(j-1)}$, $j = 1, 2, \ldots, \phi^{(0)} \equiv \phi$. It is known [1], that if A is compact then there exists an integer N such that $(**)$ will not be solvable for $j > N$. In this case vectors $\phi^{(1)}, \ldots, \phi^{(N)}$ are called the root vectors associated with the pair (λ, ϕ), $(\phi, \phi^{(1)}, \ldots, \phi^{(N)})$ is called the Jordan chain associated with the pair (λ, ϕ). Consider the eigenvectors $\phi_1, \ldots, \phi_q$ corresponding to the eigenvalue λ and all the root vectors associated with the pairs (λ, ϕ_p), $p = 1, \ldots, q$. The linear span of the eigen and root vectors corresponding to λ is called the root space corresponding to λ. The collection of all eigen and root vectors of A is called its root system. Let us define Riesz's basis of H with brackets. Let $\{f_j\}$ be a linearly independent system of elements of H, $\{h_j\}$ be an orthonormal basis of H, and $m_1 < m_2 < \cdots < m_j \longrightarrow \infty$ be a

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