BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 5, Number 2, September 1981

0002-9904/81/0000-0409/$02.00

# BOOK REVIEWS

*Permutation groups and combinatorial structures*, by N. L. Biggs and A. T. White, London Mathematical Society Lecture Note Series No. 33, Cambridge Univ. Press, New York and London, 1979, viii + 140 pp., $13.95.

The close relationships between group theory and structural combinatorics go back well over a century. Given a combinatorial object, it is natural to consider its automorphism group. Conversely, given a group, there may be a nice object upon which it acts. If the group is given as a group of permutations of some set, it is natural to try to regard the elements of that set as the points of some structure which can be at least partially visualized.

For example, in 1861 Mathieu **[8]**, **[9]** discovered five multiply transitive permutation groups. These were constructed as groups of permutations of 11, 12, 22, 23 or 24 points, by means of detailed calculations. In a little-known 1931 paper of Carmichael **[5]**, they were first observed to be automorphism groups of exquisite finite geometries. This fact was rediscovered soon afterwards by Witt **[11]**, who provided direct constructions for the groups and then the geometries. It is now more customary to construct first the designs, and then the groups, using projective planes (as in Lüneburg **[7]**) or codes (as in Cameron **[2]**, or Cameron and van Lint **[3]**). This change of point of view should be compared with the corresponding phenomenon in the case of the classical groups: they are now regarded as groups of linear transformations acting on a vector space, rather than as groups of matrices to be laboriously multiplied, inverted and conjugated.

In order to see more precisely how groups and combinatorial objects are related, consider a finite group $G$ acting transitively on a set $X$; the elements of $X$ will be called "points". Assume for the moment that $G$ is 2-transitive. Take any subset $B$ of $X$ such that $2 \leqslant |B| < |X|$, and form all the distinct images of $B$ by the elements of $G$. Any two images have the same size. Since any pair of points can be moved to any other pair by a suitable element of $G$, the number of images containing a pair is constant. Thus, this is an example of a *design* (or "balanced incomplete block design" in the statistical literature): $X$ is its set of points, and the images of $B$ are its "blocks". Designs play an important role in combinatorics, even when no group is present.

Of course, the above construction is much too general to give useful information about the group: $B$ was chosen arbitrarily, and need not have much to do with the way $G$ acts on $X$. Careful choices must be made. For example, if $G$ is the collineation group of a finite projective space, the most natural choices for $B$ are the subspaces of that space. The hindsight provided by the determination of all finite simple groups tells us that, if $G$ is any 2-transitive group other than the alternating or symmetric group on $X$, then very good choices exist for $B$. However, there is no uniform choice for $B$ which is guaranteed to reflect interesting properties of $G$.