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A REGULATOR FOR CURVES VIA THE HEISENBERG GROUP

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0. In this note we present a variation on a construction of P. Deligne [4] of the regulator map for K_2 of algebraic curves. This map, which may be viewed as an analog of the classical regulator map for the group of units in an algebraic number field, was first found for elliptic curves with complex multiplication by Spencer Bloch [2, 3]. Deligne's method involves associating, to every pair of invertible holomorphic functions f, g on a Riemann surface X, a holomorphic line bundle with connection on X, satisfying symbol properties. The new aspect of our construction is the use of a 'universal' line bundle with connection on $\mathbf{C}^* \times \mathbf{C}^*$ coming from a certain quotient M of the complex three-dimensional Heisenberg group H. We were led naturally to M while considering the transformation properties of the composite of dilogarithm with the elliptic modular function λ. Then Bloch suggested using it to define the regulator. The canonical way of defining the connection on M evolved in conversation with him.

The main interest in this is the conjectural relationship to special values of zeta functions of curves over $\mathbf{Q}$. Some progress along these lines has been made by Bloch [2] in the case of elliptic curves, and by A. Beilinson [1] in the case of modular curves. A detailed exposition of these matters, starting from the method of this note and with some additions and machine calculations, will at some future date be published jointly with Bloch.

1. Let $V = \mathbf{C}^2$, $U = (2\pi i\mathbf{Z})^2 \subset V$, B: a bilinear form on V such that $B(U \times U) \subset (2\pi i)^2\mathbf{Z}$, $e[y] = \exp[(2\pi i)^{-1}y]$, and $H_B = V \times \mathbf{C}^*$ with multiplication $(v; z) *_B (v'; z') = (v + v'; zz'e[B(v, v')])$. The left action of U on H_B, given by $(u, (v; z)) \longrightarrow (u + v; ze[B(u, v)])$, yields the identity $e[B(u + u', v)] = e[B(u, u' + v)]e[B(u', v)] \in \mathbf{C}^*$. Thus $u \longrightarrow e[B(u, -)]$ is a 1-cocycle for U with coefficients in $H^0(V, \mathcal{O}_V^*)$ determining the class of the line bundle associated with the principal $\mathbf{C}^*$-bundle $M_B \stackrel{\text{def}}{=} U\backslash H_B \xrightarrow{\phi} \mathbf{C}^* \times \mathbf{C}^* \simeq U\backslash V$, where $\phi((v_1, v_2); z) = (e^{-v_1}, e^{v_2})$ for all $(v_1 v_2)$ in V and z in $\mathbf{C}^*$. Set B_0: the form with matrix $\left(\begin{smallmatrix}0&1\\0&0\end{smallmatrix}\right)$, and write $H = H_{B_0}$, $M = M_{B_0}$, etc.

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