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HARDY'S INEQUALITY AND THE LITTLEWOOD CONJECTURE

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Let $\mathbf{T}$ be the circle group, $\mathbf{Z}$ the additive group of integers, $\mathbf{C}$ the complex numbers and $M(\mathbf{T})$ the customary convolution algebra of Borel measures on $\mathbf{T}$; for $\mu \in M(\mathbf{T})$ and $n \in \mathbf{Z}$

$$\hat{\mu}(n) = \int_{\mathbf{T}} e^{-in\theta} d\mu(\theta).$$

If $\mu \in M(\mathbf{T})$ and $\hat{\mu}(n) = 0$ for all $n < 0$, we say μ is of *analytic type*; Hardy's inequality [4, p. 70] states that for all measures μ of analytic type

$$\sum_1^\infty \frac{|\hat{\mu}(k)|}{k} \leqslant \pi \|\mu\|.$$

The purpose of this note is to announce the following generalization of Hardy's inequality:

THEOREM. *There is a real number $C > 0$ such that given any $F = \{n_1 < n_2 < \cdots < n_N\} \subset \mathbf{Z}$ and $a_j \in \mathbf{C}$, $|a_j| = 1$ $(j = 1, 2, \ldots, N)$, there corresponds a trigonometric polynomial t on $\mathbf{T}$ satisfying*

(i) $\|t\|_\infty \leqslant 1$;

(ii) $|\hat{t}(n_j)| \geqslant C/j$ $(j = 1, 2, \ldots, N)$;

(iii) $|\operatorname{sgn} \hat{t}(n_j) - a_j| < 2/5$ $(j = 1, 2, \ldots, N)$.

A simple convolution product argument now establishes the Littlewood conjecture [3] on the L^1 norm of exponential sums.

COROLLARY. *If $p(\theta) = \Sigma_{k=1}^N c_k e^{in_k\theta}$ where $\{n_1 < n_2 < \cdots < n_N\} \subset \mathbf{Z}$ and $|c_k| \geqslant 1$ $(k = 1, 2, \ldots, N)$, then*

$$\|p\|_1 \geqslant \frac{3C}{5} \log N.$$

Our proofs are based on some arguments in [5] which were inspired by the fundamental work of P. Cohen [1] and J. Fournier [2]; detailed proofs will appear in [6].

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