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THE EXISTENCE AND STABILITY OF MULTI-DIMENSIONAL SHOCK FRONTS

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We announce a proof of the short-time existence of a particularly interesting class of discontinuous weak solutions for the $m \times m$ system of hyperbolic conservation laws in several space dimensions,

$$\frac{\partial u}{\partial t} + \sum_{j=1}^{N} \frac{\partial}{\partial x_j} F_j(u) = 0, \qquad t > 0, \tag{1}$$
$$u(x, 0) = u_0(x),$$

where $x = (x_1, \ldots, x_n) \in R^N$, $u = {}^t(u_1, \ldots, u_m)$, and the $F_j(u)$ are smooth nonlinear mappings from R^m into R^m with $A_j(u) = \partial F_j/\partial u$ the corresponding Jacobian matrices. The important physical examples in gas dynamics, shallow water theory, and magnetofluid dynamics are all hyperbolic systems with the form in (1).

The only previous general existence results for systems of conservation laws in several space dimensions are due to Kato [2] who proved the local existence for solutions of (1) when $u_0(x)$ is a sufficiently smooth function. For discontinuous solutions in multi-dimensions, the only previous results for gas dynamics are the explicit spherically symmetric shock fronts described in [1].

Under suitable structural assumptions, in [4], we prove the short-time existence of *shock front* solutions for the systems in (1). These are piecewise smooth weak solutions of (1) with the following intuitive structure: There are a smooth hypersurface, $S(t)$, defined in (x, t) space for $t \geqslant 0$, noncharacteristic for (1), with space-time normal, (n_t, n_x), and two smooth vectors functions, $u^+(x, t)$ and $u^-(x, t)$, defined on respective domains, G^+ and G^-, on either side of this hypersurface so that

$$\begin{aligned} &\frac{\partial u^+}{\partial t} + \sum_{j=1}^{N} A_j(u^+) \frac{\partial u^+}{\partial x_j} = 0 \quad \text{in } G^+, \\ &\frac{\partial u^-}{\partial t} + \sum_{j=1}^{N} A_j(u^-) \frac{\partial u^-}{\partial x_j} = 0 \quad \text{in } G^-. \end{aligned} \tag{2}$$

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