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NEW CONNECTION METHOD ACROSS MORE GENERAL TURNING POINTS[1]

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The basic oscillator or Schroedinger equation

$$\epsilon^2 d^2w/dz^2 + q^2 w(z) = 0, \qquad \epsilon \downarrow 0, \tag{1}$$

with $q(z)$ analytic, except for a root or singular point at $z = 0$, poses a well-known connection problem. *WKBJ* or *L-G* approximations

$$\begin{gathered} w \sim Aq^{-1/2}\exp(i\xi/\epsilon) + Bq^{-1/2}\exp(-i\xi/\epsilon), \\ dw/z \sim (iq^{1/2}/\epsilon)[A\ \exp(i\xi/\epsilon) - B\ \exp(-i\xi/\epsilon)] \end{gathered} \tag{2}$$

with

$$\xi = \int_0^z q(s)\ ds \tag{3}$$

and constants A, B are valid in suitable regions, but not in a neighborhood of the transition point. The constants A_l, B_l in (2) valid to the left of such a point generally differ from those, A_r, B_r, valid to the right. Langer [1] found one constant pair in terms of the other for the class of "fractional turning points" at which $z^{-\nu}q(z)$ is analytic for real $\nu > -1$. But, if branch points, even of infinite order, be thus admitted, why not logarithms of z and still other branch points?

The reason lies in his "central connection" method by uniform approximation even near $z = 0$. For fractional turning points such approximands to $w(z)$ are furnished by Bessel functions, but for logarithmic turning points they are necessarily less tractable. Uniform approximation, however, is not a prerequisite for connection, nor is it needed for the most important physical applications, e.g., in scattering.

A new method will now be summarized which solves the problem for a more general class of equations (1) characterized (a precise statement is found in the Appendix) by

$$\varphi(\xi) := \tfrac{1}{2}q^{-2}dq/dz = \xi^{-1}(\gamma + o(1)) \quad \text{as } \xi \longrightarrow 0 \tag{4}$$

with real $\gamma < \frac{1}{2}$. This includes the fractional turning points, where $\gamma = \frac{1}{2}\nu/(1 + \nu)$ ($<\frac{1}{2}$ since $\nu > -1$), and also logarithmic turning points, where $q(z) \sim z^\nu(\log z)^\mu$

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