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BOOK REVIEWS

Projective geometries over finite fields, by J. W. P. Hirschfeld, Oxford Mathematical Monographs, Clarendon Press, Oxford, 1979, xii + 474 pp., $45.00.

It is a commonly held misconception that little research in the area of finite Desarguesian geometry is going on at present. While it is true that classical geometry as a discipline has been far less in vogue than it was at the beginning of this century, and that a great deal of modern research concentrates on non-Desarguesian planes, the classical case has not been ignored by all. Finite geometry in particular, inspired by the resurgence of combinatorial theory, very active work in finite algebra, and its own intrinsic appeal, has been given much attention.

Classically, $PG(n, K)$, the projective space of finite dimension n over a field K, is developed from a set of postulates, as for example in the well-known and still useful 1910 volume by Veblen and Young (*Projective geometry*, Blaisdell, New York). Nowadays, with linear algebra at hand and well developed, the concept can be introduced very quickly: Let V be the vector space of dimension $n + 1$ over K. Then $PG(n, K)$ is an incidence structure consisting of subspaces of dimension m $(0 \leqslant m < n)$, which are simply $(m + 1)$-dimensional subspaces of V; incidence is defined as inclusion. Subspaces of dimension 0, 1 and 2 are called points, lines and planes respectively. A subspace of maximum dimension $n - 1$ is called a hyperplane. Characteristic properties of projective geometry are easily established; for example, when $n = 2$ any two distinct lines in the projective plane $PG(2, K)$ intersect because any two distinct two-dimensional subspaces in a three-dimensional vector space share a one-dimensional subspace. One proceeds quickly to a study of collineations in $PG(n, K)$, these being nonsingular semilinear transformations in V. The introduction of sesquilinear forms into V, when applied to $PG(n, K)$, reproduces the classical concept of correlations (point-hyperplane correspondences), leading naturally to polarities and a study of quadric surfaces.

Projective geometry received a great deal of attention in the nineteenth and early twentieth centuries, with the concentration being almost exclusively on the geometry over the real and complex fields. The problems considered were mainly such as could be handled by methods of real and complex analysis, although they were in fact often more elegantly treated by synthetic means. Thus in addition to basic properties such as the theorems of Desargues and Pappus, with their consequences, attention was centred on conics and quadric surfaces, and generalizations thereof to curves and surfaces of higher order. In real projective geometry, theorems on order and continuity were of course very important.