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SMOOTH BOUNDED STRICTLY AND WEAKLY PSEUDOCONVEX DOMAINS CANNOT BE BIHOLOMORPHIC

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There is no Riemann mapping theorem in the theory of functions of several complex variables; nor is there a Riemann nonmapping theorem. In fact, until recently, it was not known whether it is possible for a smooth bounded strictly pseudoconvex domain to be biholomorphically equivalent to a smooth bounded weakly pseudoconvex domain. The paper [1] answers this question in the negative by proving

THEOREM 1. *If D_2 is a smooth bounded pseudoconvex domain, and D_1 is a smooth bounded domain whose $\bar{\partial}$-Neumann problem satisfies global regularity estimates, then biholomorphic mappings between D_1 and D_2 extend smoothly to the boundary.*

In particular, it is known (Kohn [6]) that the $\bar{\partial}$-Neumann problem satisfies global regularity estimates in smooth bounded strictly pseudoconvex domains. Hence, a biholomorphic mapping between a smooth bounded strictly pseudoconvex domain and a smooth bounded weakly pseudoconvex domain would extend smoothly to the boundary. Since strict pseudoconvexity is preserved under biholomorphic mappings which extend to be C^2 up to the boundary, both domains must be strictly pseudoconvex.

Other domains for which the $\bar{\partial}$-Neumann problem is known to satisfy global regularity estimates include smooth bounded weakly pseudoconvex domains with real analytic boundaries [7], [2] and certain domains of finite type [7].

The proof of Theorem 1 exploits the transformation rule for the Bergman projection. If P_i denotes the Bergman orthogonal projection of $L^2(D_i)$ onto its subspace of holomorphic functions, $i = 1, 2$, and if $f: D_1 \longrightarrow D_2$ is a biholomorphic mapping, then

$$P_1(u \cdot (\phi \circ f)) = u \cdot ((P_2\phi) \circ f)$$

where $u = \mathrm{Det}[f']$ and $\phi \in L^2(D_2)$. It is possible to construct functions ϕ which vanish to arbitrarily high order on bD_2 such that $P_2\phi \equiv 1$. If ϕ is such a function which vanishes to a high enough order on bD_2 to make $u \cdot (\phi \circ f)$ smooth up to the boundary, then u is the projection of a function which is smooth up to the

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