BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 4, Number 1, 1981

ON THE LOCAL MONODROMY OF A VARIATION OF HODGE STRUCTURE

BY EDUARDO CATTANI[1] AND AROLDO KAPLAN[1]

Associated to a variation of polarized Hodge structure there is a period mapping $\psi: S \longrightarrow \Gamma\backslash D$, where S is the parameter space and $\Gamma\backslash D$ denotes the corresponding modular variety of polarized Hodge structures (the primary example to keep in mind is that arising from a family of smooth projective varieties parametrized by S) [3], [4]. The local study of the singularities of ψ ([5]) reduces to the case when $S = (\Delta^*)^l \times \Delta^m$, a product of punctured disks and disks.

Given a lifting $\widetilde{\psi}: U^l \times \Delta^m \longrightarrow D$ (U = upper half-plane) of ψ to the universal covering of S there are monodromy transformations $\gamma_1, \ldots, \gamma_l \in \Gamma$ such that

$$\begin{aligned}\widetilde{\psi}(z_1, \ldots, z_i + 1, \ldots, z_l; w_1, \ldots, w_m)\\ = \gamma_i \widetilde{\psi}(z_1, \ldots, z_i, \ldots, z_l; w_1, \ldots, w_m).\end{aligned}$$

These γ_i's, which are quasi-unipotent automorphisms of the **C**-vector space H underlying the variation, provide important invariants of the singularities of ψ. In particular, in the single variable case ($l = 1, m = 0$) a central role is played by the monodromy weight filtration $W_* = W_*(N)$ of the nilpotent transformation $N = \log \gamma_u$, where γ_u is the unipotent part of the monodromy γ. We recall [5] that, if k is the weight of the Hodge structures, $N^{k+1} = 0$ and the filtration $(0) \subseteq W_0 \subseteq \cdots \subseteq W_{2k} = H$ is uniquely characterized by the conditions $NW_j \subseteq W_{j-2}$ and $N^j: W_{k+j}/W_{k+j-1} \longrightarrow W_{k-j}/W_{k-j-1}$ is an isomorphism.

The results announced here concern the monodromy weight filtrations arising in the several variables case. The main statements–Theorems 1 and 2–were conjectured by P. Deligne [2] (cf. Conjecture 1.9.6, as well as Theorem 1.9.2 for the special geometric case). For structures of weight two they are contained in [1].

THEOREM 1. *Let $\gamma_1, \ldots, \gamma_l$ be monodromy transformations of a period mapping $\psi: (\Delta^*)^l \times (\Delta)^m \longrightarrow \Gamma\backslash D$ and N_i the logarithm of the unipotent part*

Received by the editors August 14, 1980.

1980 *Mathematics Subject Classification*. Primary 14C30, 32G20; Secondary 22E40, 32M10.

[1] Research supported in part by an NSF Grant.

Some of the work in this paper was done while the first author was visiting Leiden University under the auspicies of the Netherland Organization for the Advancement of Pure Research (Z.W.O.), and the Institute des Hautes Études Scientifiques.