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THE STABILITY OF THE BERGMAN KERNEL AND THE GEOMETRY OF THE BERGMAN METRIC

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If D is a bounded open subset of $\mathbf{C}^n$, the set $H = \{f: D \longrightarrow \mathbf{C} | f \text{ is holomorphic and } \int_D |f|^2 < +\infty\}$ is a separable infinite-dimensional Hilbert space relative to the inner product $\langle f, g\rangle = \int_D f\bar{g}$. The completeness of H can be seen from Cauchy integral estimates. Similar estimates show that for any $p \in D$ the functional $f \mapsto f(p), f \in H$, is continuous. Thus there is a unique element $K_D(z, p) \in H$ (as a function of z) such that,

$$f(p) = \int f(z)\overline{K_D(z, p)}dV(z) \quad \text{for all } f \in H.$$

The function K_D is called the Bergman kernel function. If $\{\varphi_i\}_{i=1}^{\infty}$ is an orthonormal basis for H then $K_D(z, p) = \sum_i \varphi_i(z)\overline{\varphi_i(p)}$. The convergence of the series is absolute, uniformly on compact subsets of $D \times D$. For any $z \in D$, $K_D(z, z) > 0$ and $\log K_D(z, z)$ is a real analytic function on D. The Hermitian form

$$\sum_{i,j} \frac{\partial^2}{\partial z_i \partial \bar{z}_j} \log K_D(z, z) dz_i \otimes d\bar{z}_j$$

is positive definite on D and defines a Kähler metric on D called the Bergman metric of D. Calculations using orthonormal bases show that if $F: D_1 \longrightarrow D_2$ is a biholomorphic mapping then F is an isometry of the Bergman metric of D_1 to that of D_2. Thus differential geometric methods can be used to study such mappings. The Bergman metric is usually hard to compute explicitly; but, especially in the case of C^∞ strongly pseudoconvex domains, considerable information can be obtained by indirect methods. The starting point for such investigation is the observation that $K_D(\cdot, p)$ is, in a suitable sense, the orthogonal projection on H of the delta function δ_p. This projection can be expressed in terms of the solution of the $\bar{\partial}$-Neumann problem; a set of powerful techniques is thereby brought to bear on the matter. With these techniques, extensive information about the behavior of $K_D(z, p)$ for p, z near the boundary of D has been obtained **[9]**, **[3]**.

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