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DYNAMICS OF HOROSPHERICAL FLOWS

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Let G be a reductive Lie group; that is, the adjoint representation is completely reducible. A subgroup U of G is said to be *horospherical* if there exists $g \in G$ such that

$$U = \{u \in G \mid g^j u g^{-j} \longrightarrow e \quad \text{as} \quad j \longrightarrow \infty\}$$

where e is the identity element of G. The action of a horospherical subgroup (respectively a maximal horospherical subgroup) on a homogeneous space G/Γ, where Γ is a discrete subgroup, is called a *horospherical flow* (resp. a *maximal horospherical flow*). The classical horocycle flows associated to surfaces of constant negative curvature, studied by G. A. Hedlund and other authors, can be viewed as horospherical flows on homogeneous spaces of $SL(2, \mathbf{R})$.

In the case when Γ is a *co-compact* discrete subgroup of $SL(2, \mathbf{R})$ it was proved by H. Furstenberg that the horocycle flow is uniquely ergodic; i.e. it admits a unique invariant probability measure (cf. [6]). In particular, this implies a result of G. A. Hedlund that the horocycle flow is minimal; i.e. every orbit is dense. Satisfactory generalisations of these assertions for all horospherical flows on *compact* homogeneous spaces are available in literature (cf. [5] and [7]).

The purpose of this note is to announce similar results for maximal horospherical flows on homogeneous spaces G/Γ where Γ is any lattice; i.e. Γ is a discrete subgroup such that G/Γ admits a finite G-invariant measure, but may not necessarily be compact. When G/Γ, as above, is noncompact the horospherical flow fails to be uniquely ergodic. The general task therefore is to obtain a description of all ergodic invariant measures of the horospherical flow. We prove the following.

1. THEOREM. *Let G be a reductive Lie group and Γ be a lattice in G. Let N be a maximal horospherical subgroup of G. Let σ be an N-invariant ergodic probability measure. Then there exists a connected Lie subgroup L of G and an element $x \in G$ such that* (i) *$Lx\Gamma/\Gamma$ is a closed L-orbit in G/Γ and* (ii) *σ is L invariant and $\sigma(G/\Gamma - Lx\Gamma/\Gamma) = 0$.*

It is not difficult to show that in the special case when G/Γ is compact Theorem 1 yields the known results for maximal horospherical flows.

The proof is achieved using a characterisation of the haar measure of a

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