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AN EXAMPLE OF A FIXED POINT FREE HOMEOMORPHISM OF THE PLANE WITH BOUNDED ORBITS

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In 1912 L. E. J. Brouwer proved his famous translation theorem [3] which states that if h is an orientation preserving homeomorphism of E^2 onto itself having no fixed points, then h is a translation. By a translation, Brower meant that for each x in E^2, $h^n(x) \longrightarrow \infty$ as $n \longrightarrow \pm \infty$; that is, the orbit of every point is unbounded. The question arose as to whether or not any homeomorphism of E^2 onto itself with the property that the orbits of every point is bounded must have a fixed point. This eventually became known as the bounded orbit problem [2].

In this short note we wish to announce the existence of an orientation reversing fixed point free homeomorphism h of E^2 onto itself having the property that the orbit of every point is bounded [1]. We note that the orbit of a point p is the set of all iterates $h^n(p)$, where n is an integer. The homeomorphism we construct can be briefly described as follows. On the complement of the strip $|x| < 1$, h is a reflection across the y-axis. Between the lines $x = -1$ and $x = 1$ we first define h on positive images of the arc $A = \{(x, y): |x| \leq 1 \text{ and } y = 0\}$. For all integers $m \geq 0$ and $k \geq 1$, let

$$v_{\pm m,0} = (\pm m/(m + 1), 0),$$

and

$$v_{\pm m,k} = \left(\pm m/(m + 1), \sum_{i=1}^{k} 1/(m + i)\right).$$

For all integers j and nonnegative integers k, define

$$h(v_{j,k}) = v_{(-1)^{k+1}-j,k+1}.$$

Extend h linearly on each line segment $[v_{j-1,k}, v_{j,k}]$ by defining

$$h([v_{j-1,k}, v_{j,k}]) = [h(v_{j-1,k}), h(v_{j,k})],$$

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