BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 3, Number 2, September 1980

BLOCKS OF RATIONAL REPRESENTATIONS OF A SEMISIMPLE ALGEGRAIC GROUP

BY STEPHEN DONKIN[1]

Let G be a semisimple algebraic group over an algebraically closed field k. Any rational representation of G gives rise naturally to a representation of the Lie algebra L of G. If the characteristic of k is zero then, by a classical theorem of Weyl, every finite-dimensional representation of L is completely reducible. From this, it follows that every rational representation of G is completely reducible. However, when the characteristic of k, say p, is not zero, there are always rational representations which are not completely reducible. The extent of the lack of complete reducibility is measured, in some sense, by the block theory of G.

We say that simple rational G modules M_1 and M_2 are adjacent if both M_1 and M_2 occur as composition factors of some rational indecomposable G module. A block is then an equivalence class of simple G modules under the equivalence relation generated by adjacency. We shall also, less precisely, use the expression "V belongs to the block B" to indicate that each composition factor of the rational G module V belongs to B. Suppose that $\{B_i: i \in I\}$ is the set of blocks and that V is an arbitrary rational G module. Then V has a unique G module decomposition

$$V = \sum_{i \in I} \oplus V_i$$

such that V_i is in the block B_i for each $i \in I$.

Let T be a maximal torus of G, W the corresponding Weyl group and $(\ ,\ )$ a positive definite, W-invariant, inner product on $X(T) \otimes_{\mathbf{Z}} \mathbf{R}$, where $X(T)$ is the character group of T. Assume now that G is simply connected and that the root system of G is indecomposable (a description of the blocks in this case yields easily a description in the general case). The simple rational G modules are indexed by the elements of X^+, the set of weights in $X(T)$ which are dominant relative to some fixed choice of system of positive roots of G. For an element λ of X^+ we denote by $L(\lambda)$ the simple rational G module of highest weight λ. Each simple rational G module is isomorphic to precisely one member of $\{L(\lambda): \lambda \in X^+\}$. Thus a block of rational representations of G may be identified with a subset of X^+; for λ in X^+ we denote by $B(\lambda)$ the set of dominant weights τ such that $L(\tau)$ is in the block containing $L(\lambda)$.

Received by the editors February 12, 1980 and, in revised form, April 7, 1980.
AMS (MOS) subject classifications (1970). Primary 20G05; Secondary 20G10.
[1] Research supported by the Science Research Council of Great Britain.