about what is meant by "optimal". Finally, mention should be made of the excellent collection of exercises, some of them challenging numerical projects involving access to a high speed computer.

There are not many references in the literature where one can learn of real control problems without undue strain on credibility. This book is one of them.

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Basic set theory, by Azriel Levy, Springer-Verlag, Berlin, Heidelberg, New York, 1979, xiv + 391 pp., $24.90.

Perhaps the greatest obstacle in teaching elementary set theory is the mathematical logic needed to formalize the axioms. There is nothing inherently difficult about the basic material–the theory of ordinal and cardinal numbers, and the axiom of choice–which every mathematician is expected to know. And most of the axioms of ZF, the system of Zermelo and Fraenkel used most frequently nowadays, can be stated easily and understandably in English. The exception is the axiom scheme of replacement, which when formalized looks like this

$$\forall x_1 \cdots \forall x_n(\forall x\, \forall y\, \forall z(\varphi(x, y, x_1, \ldots, x_n) \wedge \varphi(x, z, x_1, \ldots, x_n) \rightarrow y = z) \\ \rightarrow \forall a\, \exists b\, \forall w(w \in b \leftrightarrow \exists x(x \in a \wedge \varphi(x, w, x_1, \ldots, x_n)))).$$

Here φ stands for a formula of the first-order language of set theory; each such φ yields a new instance of the axiom scheme, so there are infinitely many axioms.

Now the idea behind the replacement scheme is quite simple: any correspondence carries sets to sets. The problem is how the "correspondence" is to be specified. The solution, of course, is that the correspondence must be definable from parameters in a way which could be formalized in first-order logic. Unfortunately, many students do not find this completely clear.

Nor is this the only such problem. To take another example, each instance of the principle of definition by transfinite recursion is usually quite clear, yet the formalization of the principle itself (as a theorem scheme) is often confusing.