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SOLOMON'S CONJECTURES AND THE LOCAL FUNCTIONAL EQUATION FOR ZETA FUNCTIONS OF ORDERS

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Let A be a finite dimensional semisimple K-algebra, where K is either an algebraic number field or a complete P-adic field. Let R be a Dedekind domain with quotient field K, and let Λ be an R-order in A. Louis Solomon [3], [4] introduced a zeta function

$$\zeta_\Lambda(s) = \sum_{M \subseteq \Lambda} (\Lambda : M)^{-s},$$

where M ranges over all full left ideals of Λ. The series converges for $\mathrm{Re}(s) > 1$. Here, $(\Lambda : M)$ is the number of elements in Λ/M, and s is a complex variable. For the case where $\Lambda = R$, the above is the usual Dedekind zeta function of R, namely,

$$\zeta_R(s) = \sum (R : \mathfrak{a})^{-s},$$

where $\mathfrak{a}$ ranges over all nonzero ideals of R.

Let P range over all maximal ideals of R, and let R_P, A_P, etc., denote P-adic completions. Solomon showed easily that

$$\zeta_\Lambda(s) = \prod_P \zeta_{\Lambda_P}(s),$$

and introduced a "global" zeta function $\zeta_A(s)$, which depends on A and R but not on Λ, with the property that the P-part of $\zeta_A(s)$ coincides with $\zeta_{\Lambda_P}(s)$ for almost all P. (To be explicit, this occurs whenever A_P is a direct sum of full matrix algebras over fields, and Λ_P is a maximal R_P-order in A_P.) Solomon's conjectures involve the comparison between $\zeta_A(s)$ and $\zeta_\Lambda(s)$ at arbitrary P's.

Let us place the above in the more general setting used by Solomon. Let L be a full Λ-lattice in an A-module V, and define

$$\zeta_L(s) = \sum_{M \subseteq L} (L : M)^{-s},$$

where M ranges over all full Λ-sublattices of L. To define the "global" function $\zeta_V(s)$, we start with the Wedderburn decomposition of A:

$$A = A_1 \oplus \cdots \oplus A_r, \quad A_i \cong M_{k_i}(D_i), \quad (D_i : F_i) = e_i^2, \qquad 1 \leqslant i \leqslant r,$$

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