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Elliptic systems in the plane, by W. L. Wendland, Pitman, London, San Francisco and Melbourne, 1979, xi + 404 pp., $57.50.

Because of the close relation of elliptic equations of two variables with the theory of analytic functions, with variational principles, and with some important aspects of physics and engineering, a vast amount of literature has been compiled in the last decade concerning the properties of solutions, representation theorems, boundary value theory, constructive and computational aspects. The list of authors of important papers on the theory of elliptic operators resembles a "who is who" list of mathematical analysis. Various aspects of the theory are identified with the names of L. Bers, Agmon, Douglis and Nirenberg, Hörmander, Lions and Magenes, N. Levinson, Vekua, Bitsadze, Lavrentiev, Sobolëv, F. Browder, Miranda, Fichera, Atiyah and Singer, Malgrange, H. Lewy.

In this monograph the author deliberately avoids the abstract theory of elliptic operators, and restricts himself to the study of the two-dimensional case. The problems studied are linear. A large section of the book is devoted to the study of the normal form

$$\left.\begin{aligned} u_x - v_y &= Au + Bv + c, \\ u_y + v_x &= \tilde{A}u + Bv + \tilde{c}, \end{aligned}\right. \quad \text{in } G,$$
$$u \cos \tau(s) - v \sin \tau(s) = \phi \quad \text{on } \dot{G}, \tag{1}$$

where G denotes a bounded, simply connected domain in $\mathbf{R}^2$ with a Hölder continuously differentiable, positively oriented boundary, $\dot{G} \in \mathcal{C}^{1+\alpha}$, $\alpha > 0$.

This system of equations becomes the Cauchy-Riemann system (C.R. equations) which may be concisely written $(\partial/\partial\bar{z})w = 0$ $(w = u + iv)$, if the right hand side is identically equal to zero. Because of this close relation with the C. R. equations boundary value problem of this type permit direct applications of techniques used in the theory of analytic functions, and the use of results concerning harmonic functions. The classical problem of Dirichlet and Neumann can be solved for the region G if one knows Green's function, which is given in terms of boundary data only (on $\dot{G}$). Green's function can be found if the map $\phi(z)$ is known, where ϕ maps G conformally into the unit circle. For example for the Dirichlet problem ($\Delta u = 0$ in G, u is given on $\dot{G}$) the following formulas were known almost a century ago

$$G(z, \zeta) = \frac{1}{(2\pi)} \left| \frac{(\phi(z) - \phi(\zeta))}{(1 - \phi(z)\phi(\zeta))} \right|,$$

and

$$u(z) = \int_{\dot{G}} u(\zeta) \frac{\partial G(z, \zeta)}{\partial n} \, ds.$$

The last formula indicates that the Dirichlet problem can be replaced by an equivalent integral equation, if integration over the boundary of G is replaced by an equivalent integral over the region G.