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Differenzenapproximationen partieller Anfangswertaufgaben, by R. Ansorge, Leitfäden der angewandten Mathematik und Mechanik (LAMM), Volume 45, B. G. Teubner, Stuttgart, 1978, 298 pp., DM 29.80.

Exactly thirty years ago, when I was about to develop a serious interest in some numerical aspects of partial differential equations, two well-known mathematicians gave me the benefit of their deeper insights in the form of two predictions. "Digital computing machines will never successfully compete with analog computers. Their vaunted speed is no use, since they break down all the time" was one prediction. "The role of functional analysis in the theory of partial differential equations will always remain mostly decorative. The important ideas can equally well be expressed in the language of traditional analysis" was the second statement. The quaintness of those utterances in retrospect from 1979 came vividly to my mind when I was reading this book by R. Ansorge. Such a thorough and detailed investigation into the nature of finite difference methods would not now be considered a worthwhile effort if the first prediction had been right; and the book would not begin–as it does–with two sections entitled *Function-analytic formulation of initial value problems* and *The concept of a generalized solution*, if the language and methods of Functional Analysis had not, by now, deeply penetrated all work on partial differential equations.

There have been other widespread, more specific, predictions concerning trends in the numerical analysis of partial differential equations which would have pushed finite difference methods into the background, if they were true. One was that, as the available error estimates for these methods were, by necessity, always statements on the orders of magnitude only, rather than explicit realistic inequalities, they would be increasingly regarded as unreliable and worthless. Another one was the expectation that techniques of the Galerkin type, i.e., approximations in suitably constructed finite dimensional subspaces, such as those furnished by the finite element method would completely supersede the less flexible "old-fashioned" procedure of replacing derivatives by difference quotients in a grid.

For initial value problems, at least, as distinguished from boundary value problems, it is, however, still true that difference approximations are of paramount computational interest.

In the early history of this subject the name of Lewis F. Richardson stands out [5]. His grandiose scheme of an enormous staff of pencil pushing human computers numerous enough to solve with adequate accuracy the hyperbolic